Section 6: DFAs, NFAs, and Irregularity

Task 1 – RE to NFA

a) Convert the regular expression "$(11 \cup (01)^*)00$" to an NFA using the algorithm from lecture. You may skip adding ε-transitions for concatenation if they are obviously unnecessary, but otherwise, you should *precisely* follow the construction from lecture.

b) Convert the regular expression "$((0 \cup 1)(01)^*)^*$" to an NFA using the algorithm from lecture.

Task 2 – Irregularity

a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n 1^n 0^n \ : \ n \geqslant 0\}$ is not regular.

b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n (12)^m \ : \ n \geqslant m \geqslant 0\}$ is not regular.

Task 3 – Countability

For proving countability, you must use one of the following strategies:

- Enumeration
- Dovetailing
- Subset of a countable set
- Using a surjection (onto function) $f : \mathbb{N} \to S$
- Using an injection (one-to-one function) $f : S \to \mathbb{N}$

For proving uncountability, you must show there exists no enumeration of the elements of S (there exists no surjection $f : \mathbb{N} \to S$). This can be done using diagonalization.

a) Prove that $\{3x \ : \ x \in \mathbb{N}\}$ is countable.

b) Prove that $\mathcal{P}(\mathbb{N})$ is uncountable.

c) Show that $\mathbb{N} \times \mathbb{N}$ is countable.

Hint: How did we show that the rationals were countable?

d) Show that the countable union of countable sets is countable. That is, given a collection of sets $S_1, S_2, S_2, \ldots$ such that S_i is countable for all $i \in \mathbb{N}$, show that

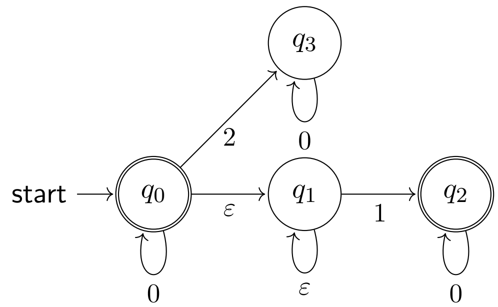
$$S = S_1 \cup S_2 \cup \cdots = \{x : x \in S_i \text{ for some } i\}$$

is countable.

Hint: Find a way of labeling the elements and see if you can apply the previous part to construct an onto function from $\mathbb{N}$ to S.

Task 4 – NFAs to DFAs

a) Convert the following NFA to a DFA for the same language:



b) Convert the following NFA to a DFA for the same language: