

Quiz Section 2: Proofs

Task 1 – Simple Formal Proofs

a) Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

Task 2 – Direct Proofs

a) Show that $\neg k \rightarrow s$ follows from $k \vee q$, $q \rightarrow r$ and $r \rightarrow s$ with a formal proof.

b) Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

Task 3 – Predicate Logic Proofs

a) Given $\forall x (T(x) \rightarrow M(x))$, prove $(\exists x T(x)) \rightarrow (\exists y M(y))$.

b) Given $\exists x (T(x) \rightarrow \forall y S(y, x))$, prove $\forall y \exists x (T(x) \rightarrow S(y, x))$.

Task 4 – English Proofs

Let domain of discourse be the integers. Consider the following claim:

$$\forall x \forall y ((\text{Even}(x) \wedge \text{Odd}(y)) \rightarrow \text{Odd}(x + y))$$

In English, this says that, for any even integer x and odd integer y , the integer $x + y$ is odd.

a) Write a **formal proof** that the claim holds.

b) Translate your formal proof to an **English proof**.

Keep in mind that your proof will be read by a *human*, not a computer, so you should explain the algebra steps in more detail, whereas some of the predicate logic steps (e.g., Elim \exists) can be skipped.