CSE 311 Section 2

Quantifiers and Proofs

Administrivia & Introductions

Announcements & Reminders

- HW1 out
  - If you think something was graded incorrectly, submit a regrade request!
  - Regrades generally will be open for a week

- HW2 Part 1 is due today **(4/16) @ 6:00 pm** on Cozy
- HW2 Part 2 is due Monday **(4/20) @ 6:00 pm** on Gradescope
  - Use a late day if you need to!
  - Gradescope: Make sure you select the pages for each question correctly
  - !! Selecting the pages after the deadline won't mark it as late

- Quiz 2 next week on Thursday (4/23)

Formal Proofs

Inference Rules to Remember

Direct Proof

$$\frac{A \Rightarrow B}{\therefore \quad A \rightarrow B}$$

Modus Ponens

$$\frac{A \qquad A \rightarrow B}{\therefore \qquad B}$$

Tautology

$$\frac{A \equiv \top}{\therefore \quad A}$$

Intro ∧

$$\frac{A \qquad B}{\therefore \quad A \wedge B}$$

Elim ∧

$$\frac{A \wedge B}{\therefore \ A \qquad B}$$

Equivalent

$$\frac{A \equiv B \qquad B}{\therefore \qquad A}$$

Intro ∨

$$\frac{A}{\therefore \ A \vee B \quad B \vee A}$$

Elim ∨

$$\frac{A \vee B \qquad \neg A}{\therefore \qquad B}$$

Proof By Cases

$$\frac{A \vee B \quad A \rightarrow C \quad B \rightarrow C}{\therefore \qquad C}$$

Relevant Rules for Problem 1a

Elim $\wedge$

$$\frac{A \wedge B}{\therefore\ A \qquad B}$$

Proof By Cases

$$\frac{A \vee B \quad A \to C \quad B \to C}{\therefore\ C}$$

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Lets get setup:

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Lets get setup:

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given
	b	

Problem 1a

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

Initial observation:

1.	$a \to \boxed{b}$	Given
2.	$c \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given

$$\boxed{b}$$

Problem 1a

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

Initial observation:
if we get a or c,
then we can get to b

1.	$a \to b$	Given
2.	$c \to b$	Given
3.	$a \vee (c \wedge d)$	Given

b

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

We can work a step back!

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given

$(a \vee c)$

b

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

What is this called?

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given

$$(a \vee c)$$

$$b$$

Problem 1a

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

What is this called?

1. $a \to b$ Given
2. $c \to b$ Given
3. $a \vee (c \wedge d)$ Given

$(a \vee c)$

b

Cases: x 1, 2

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

How can we get

$(a \vee c)$

1. $a \rightarrow b$ Given
2. $c \rightarrow b$ Given
3. $a \vee (c \wedge d)$ Given

$(a \vee c)$

b Cases: x 1, 2

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Distributivity!

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$(a \vee c) \wedge (a \vee d)$	Distributivity: 3
5.	$(a \vee c)$	
6.	b	Cases: x 1, 2

Problem 1a

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

What's missing?

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$(a \vee c) \wedge (a \vee d)$	Distributivity: 3
5.	$(a \vee c)$	
6.	b	Cases: x 1, 2

Problem 1a

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

We did it!

1.	$a \to b$	Given
2.	$c \to b$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$(a \vee c) \wedge (a \vee d)$	Distributivity: 3
5.	$(a \vee c)$	Elim $\wedge$: 4
6.	b	Cases: 5, 1, 2

Problem 1a

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

1.	$a \to b$	Given
2.	$c \to b$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$(a \vee c) \wedge (a \vee d)$	Distributivity: 3
5.	$(a \vee c)$	Elim $\wedge$: 4
6.	b	Cases: 5, 1, 2

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \to q) \land (r \to s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \to q) \land (r \to s)$	Given

Work on part b with the people around you, and then we'll go over it together!

$\neg p$

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \rightarrow q) \land (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \rightarrow q) \land (r \rightarrow s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1

$\neg p$

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \to q) \land (r \to s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \to q) \land (r \to s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\land$: 4
	$\neg p$	

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \rightarrow q) \land (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \rightarrow q) \land (r \rightarrow s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\land$: 4
6.	r	Double Negation: 5

$\neg p$

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \to q) \land (r \to s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \to q) \land (r \to s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\land$: 4
6.	r	Double Negation: 5
7.	$r \to s$	Elim $\land$: 3

$\neg p$

Problem 1b

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$, and $(p \to q) \wedge (r \to s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \vee k)$	Given
2.	$\neg q \vee \neg s$	Given
3.	$(p \to q) \wedge (r \to s)$	Given
4.	$\neg\neg r \wedge \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\wedge$: 4
6.	r	Double Negation: 5
7.	$r \to s$	Elim $\wedge$: 3
8.	s	Modus Ponens: 6, 7

$\neg p$

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \rightarrow q) \land (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \rightarrow q) \land (r \rightarrow s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\land$: 4
6.	r	Double Negation: 5
7.	$r \rightarrow s$	Elim $\land$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8

$\neg p$

Problem 1b

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$, and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \vee k)$	Given
2.	$\neg q \vee \neg s$	Given
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	Given
4.	$\neg\neg r \wedge \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\wedge$: 4
6.	r	Double Negation: 5
7.	$r \rightarrow s$	Elim $\wedge$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8
10.	$\neg s \vee \neg q$	Commutativity: 2
	$\neg p$	

Problem 1b

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$, and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \vee k)$	Given
2.	$\neg q \vee \neg s$	Given
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	Given
4.	$\neg\neg r \wedge \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\wedge$: 4
6.	r	Double Negation: 5
7.	$r \rightarrow s$	Elim $\wedge$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8
10.	$\neg s \vee \neg q$	Commutativity: 2
11.	$\neg q$	Elim $\vee$: 10, 9
	$\neg p$	

Problem 1b

b) Given $\neg(\neg r \lor k)$, $\neg q \lor \neg s$, and $(p \to q) \land (r \to s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \lor k)$	Given
2.	$\neg q \lor \neg s$	Given
3.	$(p \to q) \land (r \to s)$	Given
4.	$\neg\neg r \land \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\land$: 4
6.	r	Double Negation: 5
7.	$r \to s$	Elim $\land$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8
10.	$\neg s \lor \neg q$	Commutativity: 2
11.	$\neg q$	Elim $\lor$: 10, 9
12.	$p \to q$	Elim $\land$: 3
	$\neg p$	

Problem 1b

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$, and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \vee k)$	Given
2.	$\neg q \vee \neg s$	Given
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	Given
4.	$\neg\neg r \wedge \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\wedge$: 4
6.	r	Double Negation: 5
7.	$r \rightarrow s$	Elim $\wedge$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8
10.	$\neg s \vee \neg q$	Commutativity: 2
11.	$\neg q$	Elim $\vee$: 10, 9
12.	$p \rightarrow q$	Elim $\wedge$: 3
13.	$\neg q \rightarrow \neg p$	Contrapositive: 12
	$\neg p$	

Problem 1b

b) Given $\neg(\neg r \vee k)$, $\neg q \vee \neg s$, and $(p \rightarrow q) \wedge (r \rightarrow s)$, show that $\neg p$ holds.

1.	$\neg(\neg r \vee k)$	Given
2.	$\neg q \vee \neg s$	Given
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	Given
4.	$\neg\neg r \wedge \neg k$	De Morgan's Law: 1
5.	$\neg\neg r$	Elim $\wedge$: 4
6.	r	Double Negation: 5
7.	$r \rightarrow s$	Elim $\wedge$: 3
8.	s	Modus Ponens: 6, 7
9.	$\neg\neg s$	Double Negation: 8
10.	$\neg s \vee \neg q$	Commutativity: 2
11.	$\neg q$	Elim $\vee$: 10, 9
12.	$p \rightarrow q$	Elim $\wedge$: 3
13.	$\neg q \rightarrow \neg p$	Contrapositive: 12
14.	$\neg p$	Modus Ponens: 11, 13

Direct Proofs

Direct Proof

$$\frac{A \Rightarrow B}{\therefore A \rightarrow B}$$

Introduce an assumption like a new variable when you are conducting an experiment…

You will typically need this new assumption because your Givens alone are not sufficient

Problem 2b

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

Problem 2b

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

Just the setup:

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]

$$r \rightarrow p$$

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude if r then p?

r does not exist alone…

(2) contains r but we cannot elim or here…

$r \to p$

Problem 2b

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude if r then p?

r does not exist alone…

Could we assume r?

$$r \rightarrow p$$

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude
if r then p?

r does not exist alone…

Could we assume r?
Yes! Let's use **direct proof rule**!

$r \to p$

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

1. $p \vee \neg q$ [Given]
2. $(r \vee s) \to (q \vee s)$ [Given]
3. $\neg s$ [Given]

4.1. r [Assumption]

4 p

$r \to p$ [Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

Since we have r, can we use line 2?

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	r	[Assumption]
4	p	
	$r \to p$	[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

Since we have r, can we use line 2?
Almost! Let's create the left hand side of line 2

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	r	[Assumption]
4.2.	$r \vee s$	[$\vee$intro, 4.1]
4	p	
	$r \to p$	[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

Next: Modus Ponens!

1.	$p \vee \neg q$		[Given]
2.	$(r \vee s) \to (q \vee s)$		[Given]
3.	$\neg s$		[Given]
4.1.	r	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4	p		
	$r \to p$		[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

Next: Modus Ponens!

1.	$p \vee \neg q$		[Given]
2.	$(r \vee s) \to (q \vee s)$		[Given]
3.	$\neg s$		[Given]
4.1.	r	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4.3.	$q \vee s$	[MP 4.2, 2]	
4	p		
	$r \to p$		[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \lor \neg q$, $(r \lor s) \to (q \lor s)$, and $\neg s$.

How do we conclude p ?

We should use **q** to get to p…
How can we get **q** alone?

1.	$p \lor \neg q$	[Given]
2.	$(r \lor s) \to (q \lor s)$	[Given]
3.	$\neg s$	[Given]
4.1.	r	[Assumption]
4.2.	$r \lor s$	[$\lor$intro, 4.1]
4.3.	$q \lor s$	[MP 4.2, 2]
4	p	[$\lor$elim, 4.5, 1]
	$r \to p$	[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

We should use **q** to get to p…

use elim or!

1.	$p \vee \neg q$		[Given]
2.	$(r \vee s) \to (q \vee s)$		[Given]
3.	$\neg s$		[Given]
4.1.	r	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4.3.	$q \vee s$	[MP 4.2, 2]	
4.4.	q	[$\vee$elim, 4.3, 3]	
4	p		
	$r \to p$		[Direct Proof Rule]

Problem 2b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude p ?

We should use **q** to get to p…

use double negation!

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	r	[Assumption]
4.2.	$r \vee s$	[$\vee$intro, 4.1]
4.3.	$q \vee s$	[MP 4.2, 2]
4.4.	q	[$\vee$elim, 4.3, 3]
4.5.	$\neg\neg q$	[double negation, 4.4]
4	p	
	$r \to p$	[Direct Proof Rule]

Problem 2b

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

How do we conclude p ?

We should use **q** to get to p…

now we can use line 1!

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	r	[Assumption]
4.2.	$r \vee s$	[$\vee$intro, 4.1]
4.3.	$q \vee s$	[MP 4.2, 2]
4.4.	q	[$\vee$elim, 4.3, 3]
4.5.	$\neg\neg q$	[double negation, 4.4]
4.6.	p	[$\vee$elim, 4.5, 1]
	$r \rightarrow p$	[Direct Proof Rule]

Problem 2b

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
	4.1. r [Assumption]	
	4.2. $r \vee s$ [$\vee$intro, 4.1]	
	4.3. $q \vee s$ [MP 4.2, 2]	
	4.4. q [$\vee$elim, 4.3, 3]	
	4.5. $\neg\neg q$ [double negation, 4.4]	
	4.6. p [$\vee$elim, 4.5, 1]	
4.	$r \to p$	[Direct Proof Rule]

Predicate Logic Proofs

Predicate Logic Inference Rules

Elim ∀

$$\frac{\forall x, P(x)}{\therefore\ P(a) \text{ for } \textit{any} \text{ object } a}$$

Intro ∀

$$\frac{\text{Let } a \text{ be arbitrary} \Rightarrow P(a)}{\therefore \quad \forall x, P(x)}$$

Elim ∃

$$\frac{\exists x, P(x)}{\therefore\ P(c) \text{ for a } \textit{new} \text{ name } c}$$

Intro ∃

$$\frac{P(c) \text{ for some } c}{\therefore \quad \exists x, P(x)}$$

Formatting Reminder

There are 2 (and only 2) cases when you must indent (or subindent):

- Direct Proof
  - Direct proofs prove implications (hypothesis → conclusion)
  - Indent starting from assumption (hypothesis) all the way to conclusion
  - Every step from assumption to conclusion must be numbered
  - The line after the conclusion is the implication (assumption → conclusion), which should be de-indented, numbered, and citing direct proof (no number citation needed)

- Proving ∀x (P(x))
  - Proving ∀x (P(x)) requires proving P(x) for an arbitrary variable x
  - Introduction of arbitrary variables should not be numbered
  - Indent every step afterwards up to P(x) where x is arbitrary
  - The line after P(x) is ∀x (P(x)), which should be de-indented, numbered, and citing intro ∀ (no number citation needed)
  - If proving ∀x, ∀y (P(x,y)), you can introduce multiple arbitrary variables in one line and introduce multiple ∀ in one line

Problem 3b

b) Given $\exists x\,(T(x) \to \forall y\, S(y,x))$, prove $\forall y\, \exists x\,(T(x) \to S(y,x))$.

1. $\exists x\,(T(x) \to \forall y\, S(y,x))$ Given

Work on part b with the people around you, and then we'll go over it together!

$\forall y\, \exists x\,(T(x) \to S(y,x))$

Problem 3b

b) Given $\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$, prove $\forall y\, \exists x\,(T(x) \rightarrow S(y,x))$.

1.	$\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$	Given
2.	$T(c) \rightarrow \forall y\, S(y,c)$	Elim $\exists$: 1

$\forall y\, \exists x\,(T(x) \rightarrow S(y,x))$

Problem 3b

b) Given $\exists x\,(T(x) \to \forall y\, S(y,x))$, prove $\forall y\, \exists x\,(T(x) \to S(y,x))$.

1.	$\exists x\,(T(x) \to \forall y\, S(y,x))$	Given
2.	$T(c) \to \forall y\, S(y,c)$	Elim $\exists$: 1

Let a be arbitrary.

3.	$\forall y\, \exists x\,(T(x) \to S(y,x))$	Intro $\forall$

Problem 3b

b) Given $\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$, prove $\forall y\,\exists x\,(T(x) \rightarrow S(y,x))$.

1.	$\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$	Given
2.	$T(c) \rightarrow \forall y\, S(y,c)$	Elim ∃: 1
	Let a be arbitrary.	
3.1.1.	$T(c)$	Assumption
3.1.	$T(c) \rightarrow S(a,c)$	Direct Proof
3.	$\forall y\,\exists x\,(T(x) \rightarrow S(y,x))$	Intro ∀

Problem 3b

b) Given $\exists x\,(T(x) \to \forall y\, S(y,x))$, prove $\forall y\,\exists x\,(T(x) \to S(y,x))$.

1.	$\exists x\,(T(x) \to \forall y\, S(y,x))$	Given
2.	$T(c) \to \forall y\, S(y,c)$	Elim ∃: 1
	Let a be arbitrary.	
3.1.1.	$T(c)$	Assumption
3.1.2.	$\forall y\, S(y,c)$	Modus Ponens: 3.1.1, 2
3.1.	$T(c) \to S(a,c)$	Direct Proof
3.	$\forall y\,\exists x\,(T(x) \to S(y,x))$	Intro ∀

Problem 3b

b) Given $\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$, prove $\forall y\, \exists x\,(T(x) \rightarrow S(y,x))$.

1.	$\exists x\,(T(x) \rightarrow \forall y\, S(y,x))$	Given
2.	$T(c) \rightarrow \forall y\, S(y,c)$	Elim $\exists$: 1
	Let a be arbitrary.	
3.1.1.	$T(c)$	Assumption
3.1.2.	$\forall y\, S(y,c)$	Modus Ponens: 3.1.1, 2
3.1.3.	$S(a,c)$	Elim $\forall$: 3.1.2
3.1.	$T(c) \rightarrow S(a,c)$	Direct Proof
3.	$\forall y\, \exists x\,(T(x) \rightarrow S(y,x))$	Intro $\forall$

Problem 3b

b) Given $\exists x\,(T(x) \to \forall y\, S(y,x))$, prove $\forall y\, \exists x\,(T(x) \to S(y,x))$.

1.	$\exists x\,(T(x) \to \forall y\, S(y,x))$	Given
2.	$T(c) \to \forall y\, S(y,c)$	Elim $\exists$: 1
	Let a be arbitrary.	
3.1.1.	$T(c)$	Assumption
3.1.2.	$\forall y\, S(y,c)$	Modus Ponens: 3.1.1, 2
3.1.3.	$S(a,c)$	Elim $\forall$: 3.1.2
3.1.	$T(c) \to S(a,c)$	Direct Proof
3.2.	$\exists x\,(T(x) \to S(a,x))$	Intro $\exists$: 3.1
3.	$\forall y\, \exists x\,(T(x) \to S(y,x))$	Intro $\forall$

English Proofs

Writing a Proof (symbolically or in English)

- Don't just jump right in!

1. Look at the **claim**, and make sure you know:
   - What every word in the claim means
   - What the claim as a whole means
2. Translate the claim in predicate logic.
3. Next, write down the **Proof Skeleton**:
   - Where to **start**
   - What your **target** is
4. Then once you know what claim you are proving and your starting point and ending point, you can finally write the proof!

Helpful Tips for English Proofs

- Start by introducing your assumptions
  - Introduce variables with "let"
    - "Let x be an arbitrary prime number…"
  - Introduce assumptions with "suppose"
    - "Suppose that $y \in A \land y \notin B$…"

- When you supply a value for an existence proof, use "Consider"
  - "Consider $x = 2$…"

- **ALWAYS** state what type your variable is (integer, set, etc.)

- Universal Quantifier means variable must be arbitrary

- Existential Quantifier means variable can be specific

Problem 4

Let domain of discourse be the integers. Consider the following claim:

$$\forall x\, \forall y\, ((\mathsf{Even}(x) \wedge \mathsf{Odd}(y)) \rightarrow \mathsf{Odd}(x+y))$$

In English, this says that, for any even integer x and odd integer y, the integer $x + y$ is odd.

a) Write a **formal proof** that the claim holds.

b) Translate your formal proof to an **English proof**.

Keep in mind that your proof will be read by a *human*, not a computer, so you should explain the algebra steps in more detail, whereas some of the predicate logic steps (e.g., Elim $\exists$) can be skipped.

Problem 4a

a) Write a formal proof that $\forall x \; \forall y \; ((\text{Even}(x) \land \text{Odd}(y)) \rightarrow \text{Odd}(x + y))$

Problem 4a

a) Write a formal proof that $\forall x\ \forall y\ ((\text{Even}(x) \land \text{Odd}(y)) \rightarrow \text{Odd}(x + y))$

Let x and y be arbitrary.

1. $\forall x, \forall y, \text{Even}(x) \land \text{Odd}(y) \rightarrow \text{Odd}(x + y)$ Intro $\forall$

Problem 4a

a) Write a formal proof that $\forall x\ \forall y\ ((\text{Even}(x) \land \text{Odd}(y)) \rightarrow \text{Odd}(x + y))$

Let x and y be arbitrary.

1.1.1. $\text{Even}(x) \land \text{Odd}(y)$ Assumption

1.1. $\text{Even}(x) \land \text{Odd}(y) \rightarrow \text{Odd}(x + y)$ Direct Proof

1. $\forall x, \forall y, \text{Even}(x) \land \text{Odd}(y) \rightarrow \text{Odd}(x + y)$ Intro $\forall$

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	Even(x) ∧ Odd(y)	Assumption
1.1.2.	Even(x)	Elim ∧: 1.1.1

1.1.	Even(x) ∧ Odd(y) → Odd$(x + y)$	Direct Proof
1.	$\forall x, \forall y$, Even(x) ∧ Odd(y) → Odd$(x + y)$	Intro ∀

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	$\text{Even}(x) \land \text{Odd}(y)$	Assumption
1.1.2.	$\text{Even}(x)$	Elim $\land$: 1.1.1
1.1.3.	$\text{Odd}(y)$	Elim $\land$: 1.1.1

1.1.	$\text{Even}(x) \land \text{Odd}(y) \rightarrow \text{Odd}(x+y)$	Direct Proof
1.	$\forall x, \forall y, \text{Even}(x) \land \text{Odd}(y) \rightarrow \text{Odd}(x+y)$	Intro $\forall$

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2

1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption
1.1.2.	$\text{Even}(x)$	Elim ∧: 1.1.1
1.1.3.	$\text{Odd}(y)$	Elim ∧: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4

1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x+y)$	Direct Proof
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x+y)$	Intro ∀

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	Even(x) $\wedge$ Odd(y)	Assumption
1.1.2.	Even(x)	Elim $\wedge$: 1.1.1
1.1.3.	Odd(y)	Elim $\wedge$: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3

1.1.	Even(x) $\wedge$ Odd(y) $\rightarrow$ Odd($x + y$)	Direct Proof
1.	$\forall x, \forall y$, Even(x) $\wedge$ Odd(y) $\rightarrow$ Odd($x + y$)	Intro ∀

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption
1.1.2.	$\text{Even}(x)$	Elim ∧: 1.1.1
1.1.3.	$\text{Odd}(y)$	Elim ∧: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3
1.1.7.	$y = 2b + 1$	Elim ∃: 1.1.6

1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro ∀

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	Even(x) ∧ Odd(y)	Assumption
1.1.2.	Even(x)	Elim ∧: 1.1.1
1.1.3.	Odd(y)	Elim ∧: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3
1.1.7.	$y = 2b + 1$	Elim ∃: 1.1.6
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7

1.1.	Even(x) ∧ Odd(y) → Odd$(x + y)$	Direct Proof
1.	$\forall x, \forall y,$ Even(x) ∧ Odd(y) → Odd$(x + y)$	Intro ∀

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	Even(x) ∧ Odd(y)	Assumption
1.1.2.	Even(x)	Elim ∧: 1.1.1
1.1.3.	Odd(y)	Elim ∧: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3
1.1.7.	$y = 2b + 1$	Elim ∃: 1.1.6
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7
1.1.9.	$\exists k, x + y = 2k + 1$	Intro ∃: 1.1.8

1.1.	Even(x) ∧ Odd(y) → Odd($x + y$)	Direct Proof
1.	$\forall x, \forall y$, Even(x) ∧ Odd(y) → Odd($x + y$)	Intro ∀

Problem 4a

a) Write a formal proof that $\forall x\ \forall y\ ((\text{Even}(x) \wedge \text{Odd}(y)) \rightarrow \text{Odd}(x + y))$

Let x and y be arbitrary.

1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8
1.1.10.	$\text{Odd}(x + y)$	Undef Odd: 1.1.9
1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$

Problem 4a

a) Write a formal proof that ∀x ∀y ((Even(x) ∧ Odd(y)) → Odd(x + y))

Let x and y be arbitrary.

1.1.1.	Even(x) ∧ Odd(y)	Assumption
1.1.2.	Even(x)	Elim ∧: 1.1.1
1.1.3.	Odd(y)	Elim ∧: 1.1.1
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2
1.1.5.	$x = 2a$	Elim ∃: 1.1.4
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3
1.1.7.	$y = 2b + 1$	Elim ∃: 1.1.6
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7
1.1.9.	$\exists k, x + y = 2k + 1$	Intro ∃: 1.1.8
1.1.10.	Odd($x + y$)	Undef Odd: 1.1.9
1.1.	Even(x) ∧ Odd(y) → Odd($x + y$)	Direct Proof
1.	∀x, ∀y, Even(x) ∧ Odd(y) → Odd($x + y$)	Intro ∀

Problem 4b

b) Translate the formal proof into an English proof.

Let x and y be arbitrary. — Let x and y be arbitrary integers

1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption	
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2	
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4	
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3	
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10.	$\text{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof	
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

	Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2	
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4	
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3	
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10.	$\text{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof	
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

	Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1.	$\mathsf{Even}(x) \land \mathsf{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2.	$\mathsf{Even}(x)$	Elim $\land$: 1.1.1	
1.1.3.	$\mathsf{Odd}(y)$	Elim $\land$: 1.1.1	
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2	
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4	
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3	
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10.	$\mathsf{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1.	$\mathsf{Even}(x) \land \mathsf{Odd}(y) \to \mathsf{Odd}(x + y)$	Direct Proof	
1.	$\forall x, \forall y, \mathsf{Even}(x) \land \mathsf{Odd}(y) \to \mathsf{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

Formal proof	Rule	English proof
Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2. $\mathsf{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3. $\mathsf{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4. $\exists m, x = 2m$	Def of Even: 1.1.2	By def of even, x = 2a for some integer a
1.1.5. $x = 2a$	Elim $\exists$: 1.1.4	
1.1.6. $\exists n, y = 2n + 1$	Def of Odd: 1.1.3	
1.1.7. $y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8. $x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	
1.1.9. $\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10. $\mathsf{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y) \rightarrow \mathsf{Odd}(x + y)$	Direct Proof	
1. $\forall x, \forall y, \mathsf{Even}(x) \wedge \mathsf{Odd}(y) \rightarrow \mathsf{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2. $\mathsf{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3. $\mathsf{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4. $\exists m, x = 2m$	Def of Even: 1.1.2	By def of even, x = 2a for some integer a
1.1.5. $x = 2a$	Elim $\exists$: 1.1.4	
1.1.6. $\exists n, y = 2n + 1$	Def of Odd: 1.1.3	By def of odd, y = 2b + 1 for some integer b
1.1.7. $y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8. $x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	
1.1.9. $\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10. $\mathsf{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y) \to \mathsf{Odd}(x + y)$	Direct Proof	
1. $\forall x, \forall y, \mathsf{Even}(x) \wedge \mathsf{Odd}(y) \to \mathsf{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

	Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2	By def of even, x = 2a for some integer a
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4	
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3	By def of odd, y = 2b + 1 for some integer b
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	Thus, x + y = 2 (a + b) + 1
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	
1.1.10.	$\text{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof	
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

Formal proof	Justification	English proof
Let x and y be arbitrary.		Let x and y be arbitrary integers
1.1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2. $\mathsf{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3. $\mathsf{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4. $\exists m, x = 2m$	Def of Even: 1.1.2	By def of even, x = 2a for some integer a
1.1.5. $x = 2a$	Elim $\exists$: 1.1.4	
1.1.6. $\exists n, y = 2n + 1$	Def of Odd: 1.1.3	By def of odd, y = 2b + 1 for some integer b
1.1.7. $y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8. $x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	Thus, x + y = 2 (a + b) + 1
1.1.9. $\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	Since integers are closed under addition, a + b is an integer
1.1.10. $\mathsf{Odd}(x + y)$	Undef Odd: 1.1.9	
1.1. $\mathsf{Even}(x) \wedge \mathsf{Odd}(y) \rightarrow \mathsf{Odd}(x + y)$	Direct Proof	
1. $\forall x, \forall y, \mathsf{Even}(x) \wedge \mathsf{Odd}(y) \rightarrow \mathsf{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

Let x and y be arbitrary.			Let x and y be arbitrary integers
1.1.1.	$\text{Even}(x) \wedge \text{Odd}(y)$	Assumption	Suppose x is even and y is odd
1.1.2.	$\text{Even}(x)$	Elim $\wedge$: 1.1.1	
1.1.3.	$\text{Odd}(y)$	Elim $\wedge$: 1.1.1	
1.1.4.	$\exists m, x = 2m$	Def of Even: 1.1.2	By def of even, x = 2a for some integer a
1.1.5.	$x = 2a$	Elim $\exists$: 1.1.4	
1.1.6.	$\exists n, y = 2n + 1$	Def of Odd: 1.1.3	By def of odd, y = 2b + 1 for some integer b
1.1.7.	$y = 2b + 1$	Elim $\exists$: 1.1.6	
1.1.8.	$x + y = 2(a + b) + 1$	Algebra: 1.1.5 1.1.7	Thus, x + y = 2 (a + b) + 1
1.1.9.	$\exists k, x + y = 2k + 1$	Intro $\exists$: 1.1.8	Since integers are closed under addition, a + b is an integer
1.1.10.	$\text{Odd}(x + y)$	Undef Odd: 1.1.9	By def of odd, x + y is odd
1.1.	$\text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Direct Proof	
1.	$\forall x, \forall y, \text{Even}(x) \wedge \text{Odd}(y) \rightarrow \text{Odd}(x + y)$	Intro $\forall$	Since x and y were arbitrary, the claim holds

Problem 4b

b) Translate the formal proof into an English proof.

Let x and y be arbitrary integers. Suppose x is even and y is odd.

By the definition of even, $x = 2a$ for some integer a, and by the definition of odd, $y = 2b + 1$ for some integer b. Thus, $x + y = 2a + 2b + 1 = 2(a + b) + 1$. As integers are closed under addition, $a + b$ is an integer such that $x + y$ is odd, by definition of odd.

Since x and y were arbitrary, we have proven the desired result.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?