CSE 311 Section 1

Predicate Logic and Equivalences

Announcements & Reminders

- Sections are Graded
  - You will be graded on section participation, so please try to come ☺
  - If you cannot attend you will need to submit ALL the section problems to gradescope by 6:00 pm on the day after section

- HW1 Part 1 due today **(4/7) @ 6:00 PM** on Cozy
- HW1 Part 2 due Friday **(4/10) @ 6:00 PM** on Gradescope
  - Remember, you only have 3 late days to use throughout the quarter
  - You can use only 1 late day on any 1 assignment
  - Submitting both parts of one assignment late counts as 1 late day

- **Quiz 1 next week on Tuesday 4/14**

- Check the course website for OH times!

- Concept Checks!
  - Absolute deadline on the day after the lecture is given **@ 6:00 pm**

Task 1

Logical Equivalences

Identity

$$p \wedge \mathsf{T} \equiv p$$
$$p \vee \mathsf{F} \equiv p$$

Domination

$$p \vee \mathsf{T} \equiv \mathsf{T}$$
$$p \wedge \mathsf{F} \equiv \mathsf{F}$$

DeMorgan's Laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Idempotency

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

Commutativity

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

Double Negation

$$\neg\neg p \equiv p$$

Associativity

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Distributivity

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Contrapositive

$$p \to q \equiv \neg q \to \neg p$$

Absorption

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

Negation

$$p \vee \neg p \equiv \mathsf{T}$$
$$p \wedge \neg p \equiv \mathsf{F}$$

Law of Implication

$$p \to q \equiv \neg p \vee q$$

Task 1 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

is this true?

Task 1 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$ Yes! Truth values match!

p	q	r	$\neg p$	$(q \rightarrow r)$	$(p \vee r)$	$\neg p \rightarrow (q \rightarrow r)$	$q \rightarrow (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \vee \mathsf{T} \equiv \mathsf{T}$

$p \wedge \mathsf{F} \equiv \mathsf{F}$

DeMorgan's Laws

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Idempotency

$p \vee p \equiv p$

$p \wedge p \equiv p$

Commutativity

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

Double Negation

$\neg\neg p \equiv p$

Associativity

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Absorption

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

Negation

$p \vee \neg p \equiv \mathsf{T}$

$p \wedge \neg p \equiv \mathsf{F}$

Law of Implication

$p \to q \equiv \neg p \vee q$

Task 1 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$$\neg p \to (q \to r) \quad \equiv \quad \neg\neg p \vee (q \to r) \qquad \text{Law of Implication}$$

$$\equiv \quad \neg q \vee (p \vee r)$$

$$\equiv \quad q \to (p \vee r) \qquad \text{Law of Implication}$$

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$$
\begin{aligned}
\neg p \to (q \to r) &\equiv \neg\neg p \vee (q \to r) && \text{Law of Implication} \\
&\equiv p \vee (q \to r) && \text{Double Negation} \\
&\equiv \neg q \vee (p \vee r) \\
&\equiv q \to (p \vee r) && \text{Law of Implication}
\end{aligned}
$$

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$\neg q \vee (p \vee r)$	
	$\equiv$	$q \to (p \vee r)$	Law of Implication

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$\neg q \vee (p \vee r)$	
	$\equiv$	$q \to (p \vee r)$	Law of Implication

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$(\neg q \vee p) \vee r$	Commutativity
	$\equiv$	$\neg q \vee (p \vee r)$	
	$\equiv$	$q \to (p \vee r)$	Law of Implication

Task 1 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$(\neg q \vee p) \vee r$	Commutativity
	$\equiv$	$\neg q \vee (p \vee r)$	Associativity
	$\equiv$	$q \to (p \vee r)$	Law of Implication

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv p \vee q$

Is this true?

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv p \vee q$

Yes! Truth values match!

q	p	r	$\neg p$	$q \vee p$	$r \vee \neg p$	$\neg(q \vee p)$	$\neg(r \vee \neg p)$	$\neg(q \vee p) \vee \neg(r \vee \neg p)$	$\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p$	$p \vee q$
T	T	T	F	T	T	F	F	F	T	T
T	T	F	F	T	F	F	T	T	T	T
T	F	T	T	T	T	F	F	F	T	T
T	F	F	T	T	T	F	F	F	T	T
F	T	T	F	T	T	F	F	F	T	T
F	T	F	F	T	F	F	T	T	T	T
F	F	T	T	F	T	T	F	T	F	F
F	F	F	T	F	T	T	F	T	F	F

Task 1 – Equivalences

We did the start of the proof for you. Work on part c with the people around you, and then we'll go over it together!

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

Identity

$$p \wedge \mathsf{T} \equiv p$$
$$p \vee \mathsf{F} \equiv p$$

Domination

$$p \vee \mathsf{T} \equiv \mathsf{T}$$
$$p \wedge \mathsf{F} \equiv \mathsf{F}$$

DeMorgan's Laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Idempotency

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

Commutativity

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

Double Negation

$$\neg\neg p \equiv p$$

Associativity

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Distributivity

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Absorption

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

Negation

$$p \vee \neg p \equiv \mathsf{T}$$
$$p \wedge \neg p \equiv \mathsf{F}$$

Law of Implication

$$p \rightarrow q \equiv \neg p \vee q$$

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$ Distributivity

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$ Distributivity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$ Commutativity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$ Commutativity

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$ Distributivity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$ Commutativity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$ Commutativity

$\equiv ((p \vee p) \vee q) \wedge (p \vee (\neg p \vee r))$ Associativity

$\equiv ((p \vee p) \vee q) \wedge ((p \vee \neg p) \vee r)$ Associativity

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$ Distributivity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$ Commutativity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$ Commutativity

$\equiv ((p \vee p) \vee q) \wedge (p \vee (\neg p \vee r))$ Associativity

$\equiv ((p \vee p) \vee q) \wedge ((p \vee \neg p) \vee r)$ Associativity

$\equiv (p \vee q) \wedge ((p \vee \neg p) \vee r)$ Idempotency

$\equiv p \vee q$

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p$

$\equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Contrapositive
$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Law of Implication
$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Double Negation
$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$	De Morgan's Law
$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$	Double Negation
$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$	Distributivity
$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$	Commutativity
$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$	Commutativity
$\equiv ((p \vee p) \vee q) \wedge (p \vee (\neg p \vee r))$	Associativity
$\equiv ((p \vee p) \vee q) \wedge ((p \vee \neg p) \vee r)$	Associativity
$\equiv (\mathrm{p} \vee q) \wedge ((p \vee \neg p) \vee r)$	Idempotency
$\equiv (\mathrm{p} \vee q) \wedge (\mathrm{T} \vee r)$	Negation
$\equiv p \vee q$	

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p$

$\equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Contrapositive
$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Law of Implication
$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$	Double Negation
$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$	De Morgan's Law
$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$	Double Negation
$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$	Distributivity
$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$	Commutativity
$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$	Commutativity
$\equiv ((p \vee p) \vee q) \wedge (p \vee (\neg p \vee r))$	Associativity
$\equiv ((p \vee p) \vee q) \wedge ((p \vee \neg p) \vee r)$	Associativity
$\equiv (\mathrm{p} \vee q) \wedge ((p \vee \neg p) \vee r)$	Idempotency
$\equiv (\mathrm{p} \vee q) \wedge (\mathrm{T} \vee r)$	Negation
$\equiv (\mathrm{p} \vee q) \wedge \mathrm{T}$	Domination
$\equiv p \vee q$	

Task 1 – Equivalences

c) $\neg(q \vee p) \vee \neg(r \vee \neg p) \rightarrow p \equiv \neg p \rightarrow \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Contrapositive

$\equiv \neg\neg p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Law of Implication

$\equiv p \vee \neg(\neg(q \vee p) \vee \neg(r \vee \neg p))$ Double Negation

$\equiv p \vee \neg\neg((q \vee p) \wedge (r \vee \neg p))$ De Morgan's Law

$\equiv p \vee ((q \vee p) \wedge (r \vee \neg p))$ Double Negation

$\equiv (p \vee (q \vee p)) \wedge (p \vee (r \vee \neg p))$ Distributivity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (r \vee \neg p))$ Commutativity

$\equiv (p \vee (p \vee q)) \wedge (p \vee (\neg p \vee r))$ Commutativity

$\equiv ((p \vee p) \vee q) \wedge (p \vee (\neg p \vee r))$ Associativity

$\equiv ((p \vee p) \vee q) \wedge ((p \vee \neg p) \vee r)$ Associativity

$\equiv (\mathrm{p} \vee q) \wedge ((p \vee \neg p) \vee r)$ Idempotency

$\equiv (\mathrm{p} \vee q) \wedge (\mathrm{T} \vee r)$ Negation

$\equiv (\mathrm{p} \vee q) \wedge \mathrm{T}$ Domination

$\equiv p \vee q$ Identity

Task 2

Predicates & Quantifiers

- **Predicate**: A function that outputs booleans.
  - Red(x) outputs true if x is red
  - EqualTo(x, y) outputs true if x is equal to y
- **Domain of Discourse**: The types of things that can be inputs to a predicate.
  - Integers, real numbers, colors, mammals, students, etc
- **Quantifiers**
  - Universal Quantifier $\forall x$ P(x): For all x, P(x) is true. *(In latex, \forall)*
  - Existential Quantifier $\exists x$ P(x): There exists an x, such that P(x) is true. *(In latex, \exists)*
- **Domain Restrictions**
  - When restricting $\forall$, add the domain restriction as the hypothesis of an implication
  - When restricting $\exists$, AND the domain restriction with the statement

Task 2 – Predicates

CS(x) returns true if and only if x majors in CS

CE(x) returns true if and only if x majors in CE

CSE(y) returns true if and only if y is a CSE class

MATH(y) returns true if and only if y is a MATH class

Wants(x,y) returns true if and only if x wants to take y

Likes(x,y) returns true if and only if x likes y

HasToTake(x,y) returns true if and only if x has to take y

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

b) $\exists x\, (CS(x) \wedge \exists y (CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

Task 2 – Predicates

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

b) $\exists x\, (CS(x) \wedge \exists y (CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

Task 2 – Predicates

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

There is no student that majors in both CS and CE.

b) $\exists x\, (CS(x) \wedge \exists y (CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

Task 2 – Predicates

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

There is no student that majors in both CS and CE.

b) $\exists x\, (CS(x) \wedge \exists y(CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

There is a CS student who likes a CSE class they don't have to take.

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

Task 2 – Predicates

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

There is no student that majors in both CS and CE.

b) $\exists x\, (CS(x) \wedge \exists y (CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

There is a CS student who likes a CSE class they don't have to take.

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

All CE students have a MATH class they have to take.

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

Task 2 – Predicates

a) $\neg \exists x\, (CS(x) \wedge CE(x))$

There is no student that majors in both CS and CE.

b) $\exists x\, (CS(x) \wedge \exists y(CSE(y) \wedge \neg HasToTake(x,y) \wedge Likes(x,y)))$

There is a CS student who likes a CSE class they don't have to take.

c) $\forall x\, (CE(x) \rightarrow \exists y\, (MATH(y) \wedge HasToTake(x,y)))$

All CE students have a MATH class they have to take.

d) $\exists x\, ((CS(x) \vee CE(x)) \wedge \forall y\, (CSE(y) \rightarrow Wants(x,y)))$

There is a student who majors in CS or CE and wants to take all CSE classes.

Task 3

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

a) Every user has access to an electronic mailbox.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

a) Every user has access to an electronic mailbox.

Let the domain be users and mailboxes. Let $\texttt{User}(x)$ be "x is a user", let $\texttt{Mailbox}(y)$ be "y is a mailbox", and let $\texttt{Access}(x, y)$ be "x has access to y".

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

a) Every user has access to an electronic mailbox.

Let the domain be users and mailboxes. Let $\texttt{User}(x)$ be "x is a user", let $\texttt{Mailbox}(y)$ be "y is a mailbox", and let $\texttt{Access}(x, y)$ be "x has access to y".

$$\forall x \, (\texttt{User}(x) \rightarrow (\exists y \, (\texttt{Mailbox}(y) \wedge \texttt{Access}(x, y))))$$

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Let the domain be people and mailboxes and use $\texttt{Access}(x, y)$ as defined in the solution to part (a), and then also add $\texttt{InGroup}(x)$ for "x is in the group", and let $\texttt{SystemMailBox}$ be the name for the system mailbox. Then the translation becomes

$$\texttt{FileSystemLocked} \rightarrow \forall x \, (\texttt{InGroup}(x) \rightarrow \texttt{Access}(x, \texttt{SystemMailBox})).$$

Task 3 – Translations to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Let the domain be people in the group. Let $\texttt{CanAccessSM}(x)$ be "x has access to the system mailbox". Let p be the proposition "the file system is locked."

$$p \rightarrow \forall x \ \texttt{CanAccessSM}(x).$$

Task 4

Task 4 – Canonical Forms

```
public static boolean E(boolean a, boolean b, boolean c, boolean d) {
  if (!(a || b))
    return false;
  if (!(!a || !b))
    return false;
  if (!(a || c))
    return false;
  return (b || !d);
}
```

Or, equivalently:

```
return (a || b) && (!a || !b) && (a || c) && (b || !d);
```

Both calculate the CNF (AND of ORs) expression for

$(a \vee b) \wedge (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee \neg d)$

Task 4 – Canonical Forms

a) Write a truth table for *E*. Include columns for *a, b, c, d*, all four disjunctions, and *E*.
E calculates $(a \vee b) \wedge (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee \neg d)$.

b) Write the **canonical** DNF expression for *E*.

c) Translate your DNF expression into a new Java implementation of *E*.

Work on problem 2 with the people around you, and then we'll go over it together!

Task 4 – Canonical Forms

Write a truth table for *E*. Include columns for *a, b, c, d*, all four disjunctions, and *E*. *E* calculates $(a \vee b) \wedge (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee \neg d)$.

a	*b*	$a \vee b$	$\neg a \vee \neg b$
F	F		
F	T		
T	F		
T	T		

a	*c*	$a \vee c$
F	F	
F	T	
T	F	
T	T	

b	*d*	$b \vee \neg d$
F	F	
F	T	
T	F	
T	T	

Task 4 – Canonical Forms

Write a truth table for *E*. Include columns for *a, b, c, d*, all four disjunctions, and *E*. *E* calculates $(a \vee b) \wedge (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee \neg d)$.

a	*b*	$a \vee b$	$\neg a \vee \neg b$
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	F

a	*c*	$a \vee c$
F	F	F
F	T	T
T	F	T
T	T	T

b	*d*	$b \vee \neg d$
F	F	T
F	T	F
T	F	T
T	T	T

Task 4 – Canonical Forms

a	b	c	d	$a \vee b$	$\neg a \vee \neg b$	$a \vee c$	$b \vee \neg d$	E
F	F	F	F	F	T	F	T	F
F	F	F	T	F	T	F	F	F
F	F	T	F	F	T	T	T	F
F	F	T	T	F	T	T	F	F
F	T	F	F	T	T	F	T	F
F	T	F	T	T	T	F	T	F
F	T	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T
T	F	F	T	T	T	T	F	F
T	F	T	F	T	T	T	T	T
T	F	T	T	T	T	T	F	F
T	T	F	F	T	F	T	T	F
T	T	F	T	T	F	T	T	F
T	T	T	F	T	F	T	T	F
T	T	T	T	T	F	T	T	F

Task 4 – Canonical Forms

a	*b*	*c*	*d*	$a \vee b$	$\neg a \vee \neg b$	$a \vee c$	$b \vee \neg d$	*E*
F	F	F	F	F	T	F	T	F
F	F	F	T	F	T	F	F	F
F	F	T	F	F	T	T	T	F
F	F	T	T	F	T	T	F	F
F	T	F	F	T	T	F	T	F
F	T	F	T	T	T	F	T	F
F	T	T	F	T	T	T	T	T
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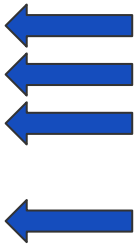
T	T	T	T	T	F	T	T	F



True Rows

Task 4 – Canonical Forms

b) Write the **canonical** DNF (OR of ANDs) expression for *E*.

a	***b***	***c***	***d***	***E***
F	T	T	F	T
F	T	T	T	T
T	F	F	F	T
T	F	T	F	T

Task 4 – Canonical Forms

b) Write the **canonical** DNF expression for *E*.

a	***b***	***c***	***d***	***E***
F	T	T	F	T
F	T	T	T	T
T	F	F	F	T
T	F	T	F	T

$$(\neg a \wedge b \wedge c \wedge \neg d)$$
$$\vee$$
$$(\neg a \wedge b \wedge c \wedge d)$$
$$\vee$$
$$(a \wedge \neg b \wedge \neg c \wedge \neg d)$$
$$\vee$$
$$(a \wedge \neg b \wedge c \wedge \neg d)$$

Task 4 – Canonical Forms

c) Translate your DNF expression into a new Java implementation of *E*.

$(\neg a \wedge b \wedge c \wedge \neg d) \vee (\neg a \wedge b \wedge c \wedge d) \vee (a \wedge \neg b \wedge \neg c \wedge \neg d) \vee (a \wedge \neg b \wedge c \wedge \neg d)$

Task 4 – Canonical Forms

c) Translate your DNF expression into a new Java implementation of *E*.

$(\neg a \wedge b \wedge c \wedge \neg d) \vee (\neg a \wedge b \wedge c \wedge d) \vee (a \wedge \neg b \wedge \neg c \wedge \neg d) \vee (a \wedge \neg b \wedge c \wedge \neg d)$

```
public static boolean E(boolean a, boolean b, boolean c, boolean d) {
  if (!a && b && c && !d)
    return true;
  if (!a && b && c && d)
    return true;
  if (a && !b && !c && !d)
    return true;
  return a && !b && c && !d;
}
```

That's All, Folks!

Thanks for coming to section this week!
Any questions?