Quiz Section 9: DFAs, NFAs, Relations

Task 1 – Good, Good, Good, Good Relations

Each part below defines a relation R on a set. For each part, first state whether R is reflexive, symmetric, antisymmetric, and/or transitive. Second, if a relation does *not* have a property, then state a counterexample. (If a relation *does* have a property, you don't need to do anything other than saying so.)

a) Let $R = \{(x, y) \ : \ x = y + 1\}$ on $\mathbb{N}$.

b) Let $R = \{(x, y) \ : \ x^2 = y^2\}$ on $\mathbb{R}$.

Task 2 – Relations

Let A be a set, and let R and S be relations on A. Suppose that R is reflexive.

a) Prove that $R \cup S$ is reflexive.

b) Prove that $R \subseteq R^2$. (Remember that R^2 is defined to be $R \circ R$.)

Task 3 – String Relations

Let $\Sigma = \{0, 1\}$. Define the relation R on Σ^* by $(x, y) \in R$ if and only if len(xy) is even. (Here xy is notation for the concatenation of the two strings x and y and len refers to the length of the string.)

Hint: In your proofs below, you may use the fact from lecture that len(xy) = len(x) + len(y).

a) Prove that R is reflexive.

b) Prove that R is symmetric.

c) Prove that R is transitive.

d) Is R antisymmetric? If so, prove it. If not, give a counterexample.

Task 4 – DFAs, Stage 1

Let $\Sigma = \{0, 1, 2, 3\}$. Construct DFAs to recognize each of the following languages.

For all states in your DFA, include "documentation" for them by describing, in English, the set of strings that *end* in that state.

a) All binary strings.

b) All strings whose digits sum to an even number.

c) All strings whose digits sum to an odd number.

Task 5 – DFAs, Stage 2

Let $\Sigma = \{0, 1\}$. Construct DFAs to recognize each of the following languages.

For all states in your DFA, include "documentation" for them by describing, in English, the set of strings that *end* in that state.

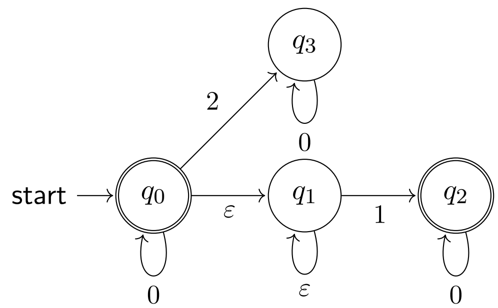
a) All strings that do not contain the substring 101.

b) All strings containing at least two 0's and at most one 1.

c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

Task 6 – NFAs

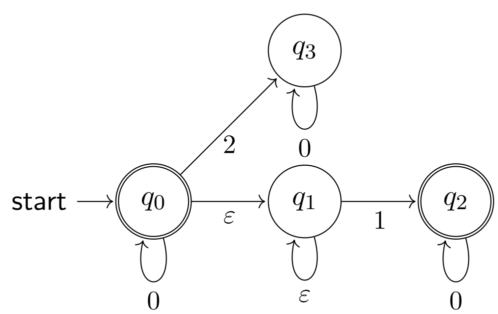
a) What language does the following NFA accept?



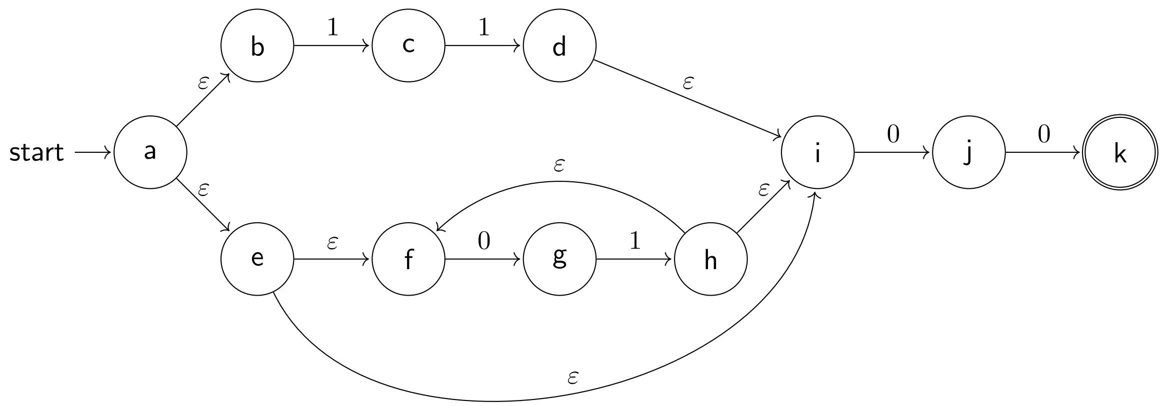
b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Task 7 – NFAs to DFAs

a) Convert the following NFA to a DFA for the same language:



b) Convert the following NFA to a DFA for the same language:



Task 8 – RE to NFA

Convert the regular expression "$(11 \cup (01)^*)00$" to an NFA using the algorithm from lecture. You may skip adding ε-transitions for concatenation if they are obviously unnecessary, but otherwise, you should *precisely* follow the construction from lecture.

Task 9 – Irregularity

a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n 1^n 0^n \ : \ n \geqslant 0\}$ is not regular.

b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n (12)^m \ : \ n \geqslant m \geqslant 0\}$ is not regular.