Quiz Section 7: Set Theory

Review

Set theory:

- $A \setminus B = \{x : x \in A \wedge x \notin B\}$. Or, equivalently, $x \in A \setminus B \leftrightarrow x \in A \wedge x \notin B$.
- $A \times B = \{(a, b) : a \in A,\ b \in B\}$. Or, equivalently, $(a, b) \in A \times B \leftrightarrow a \in A \wedge b \in B$.
- $\mathcal{P}(A) = \{B : B \subseteq A\}$. Or, equivalently, $B \in \mathcal{P}(A) \leftrightarrow B \subseteq A$.

Task 1 – How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say ∞.

a) $A = \{1, 2, 3, 2\}$

b) $B = \{\{\},\ \{\{\}\},\ \{\{\}, \{\}\},\ \{\{\}, \{\}, \{\}\},\ \ldots\}$

c) $C = A \times (B \cup \{7\})$

d) $D = \varnothing$

e) $E = \{\varnothing\}$

f) $F = \mathcal{P}(\{\varnothing\})$

Task 2 – Set Replay

Prove each of the following set identities.

a) $A \setminus B \subseteq A \cup C$ for any sets A, B, C.

b) $(A \backslash B) \backslash C \subseteq A \backslash C$ for any sets A, B, C.

c) $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

Task 3 – Set Equality

Let A and B be sets. Consider the claim: $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.
State what the claim becomes when you unroll the definition of "$=$" sets. Then, following the Meta Theorem template, write an English proof that the claim holds.

Task 4 – Power Sets

Let A and B be sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ follows from $A \subseteq B$.

Task 5 – Set = Set

Prove the following set identities. Write both a formal inference proof **and** an English proof.

1. Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \backslash B$ for any sets A, B.

2. Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.