CSE 311 Section 7

Set Theory

Announcements & Reminders

- Congrats on finishing the midterm!
  - Please don’t discuss as not everyone has taken it :)
- Homework 6 due Wednesday, February 26th @ 11:00pm
- Book One-on-Ones on the course homepage!

Set Elements

Problem 1 – How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say ∞.

a) $A = \{1, 2, 3, 2\}$

b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$

c) $C = A \times (B \cup \{7\})$

d) $D = \emptyset$

e) $E = \{\emptyset\}$

f) $F = \mathcal{P}(\{\emptyset\})$

Work this problem with the people around you, and then we'll go over it together!

Sets: Quick Review

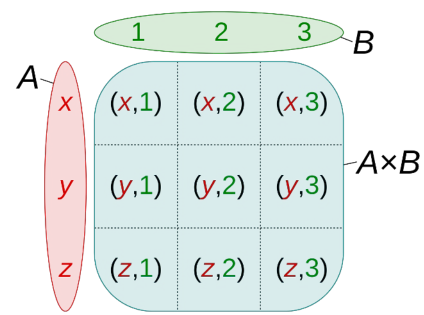
Sets

- A set is an **unordered** group of **distinct** elements
  - Set variable names are capital letters, with lower-case letters for elements

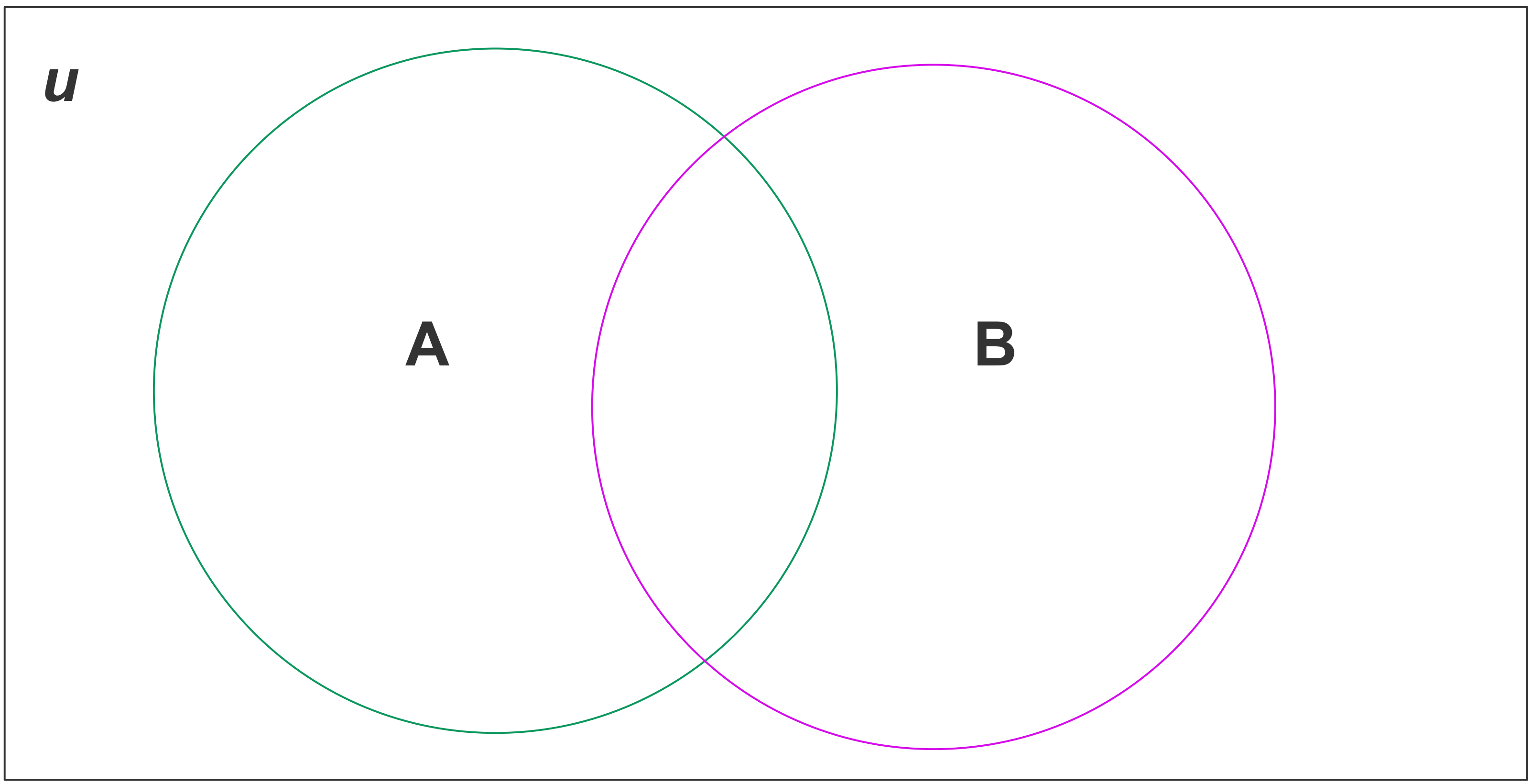
- Set Notation:
  - $a \in A$: "a is in A" or "a is an element of A"
  - $A \subseteq B$: "A is a subset of B", every element of A is also in B
  - $\emptyset$: "empty set", a unique set containing no elements
  - $\mathcal{P}(A)$: "power set of A", the set of all subsets of A including the empty set and A itself

Set Operators

- Subset: $A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$
- Equality: $A = B \equiv \forall x(x \in A \leftrightarrow x \in B) \equiv A \subseteq B \wedge B \subseteq A$
- Union: $A \cup B = \{x : x \in A \vee x \in B\}$
- Intersection: $A \cap B = \{x : x \in A \wedge x \in B\}$
- Complement: $\overline{A} = \{x : x \notin A\}$
- Difference: $A \backslash \mathrm{B} = \{x : x \in A \wedge x \notin B\}$
- Cartesian Product: $A \times B = \{(a, b) : a \in A \wedge b \in B\}$

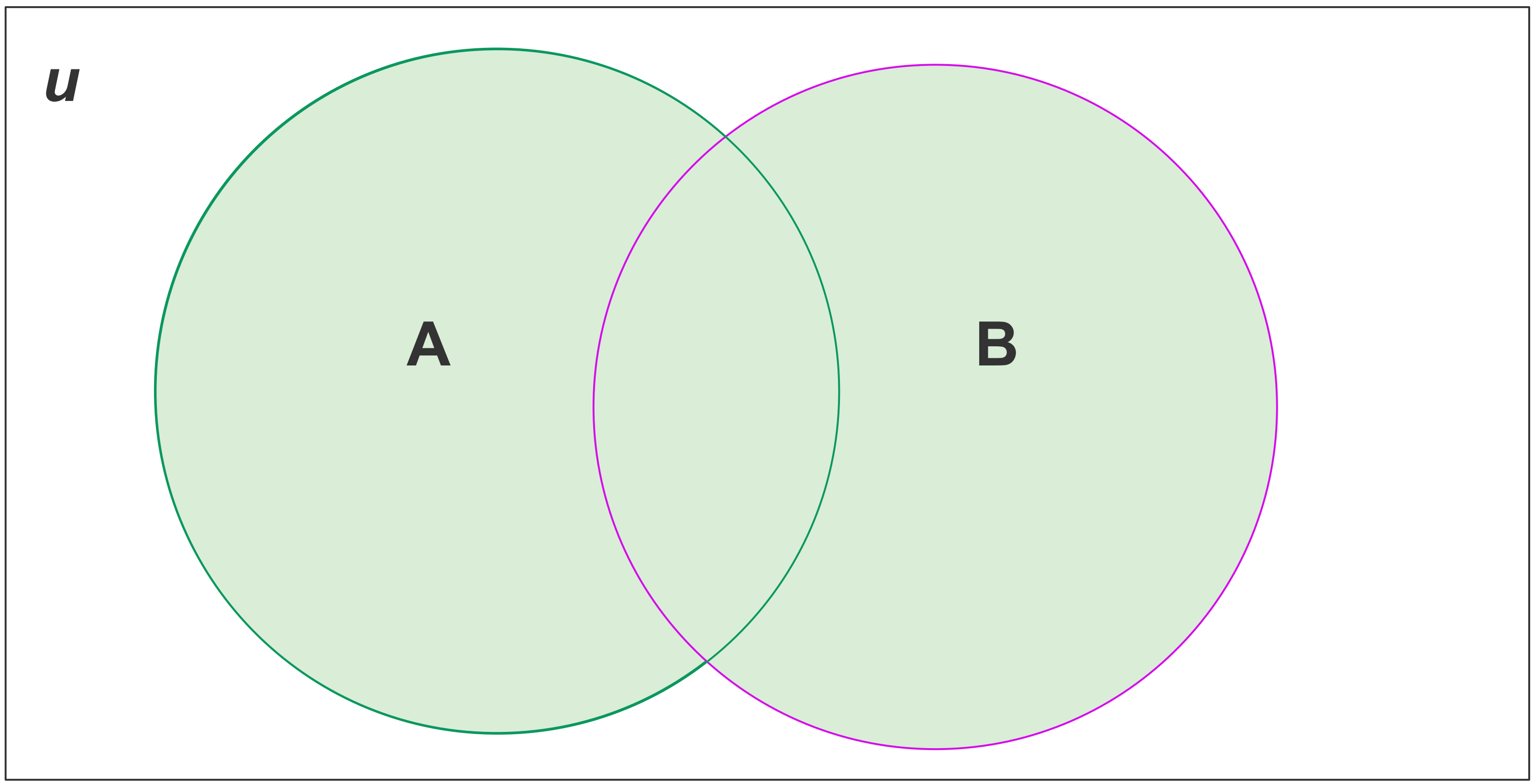


Understand Sets Visually!



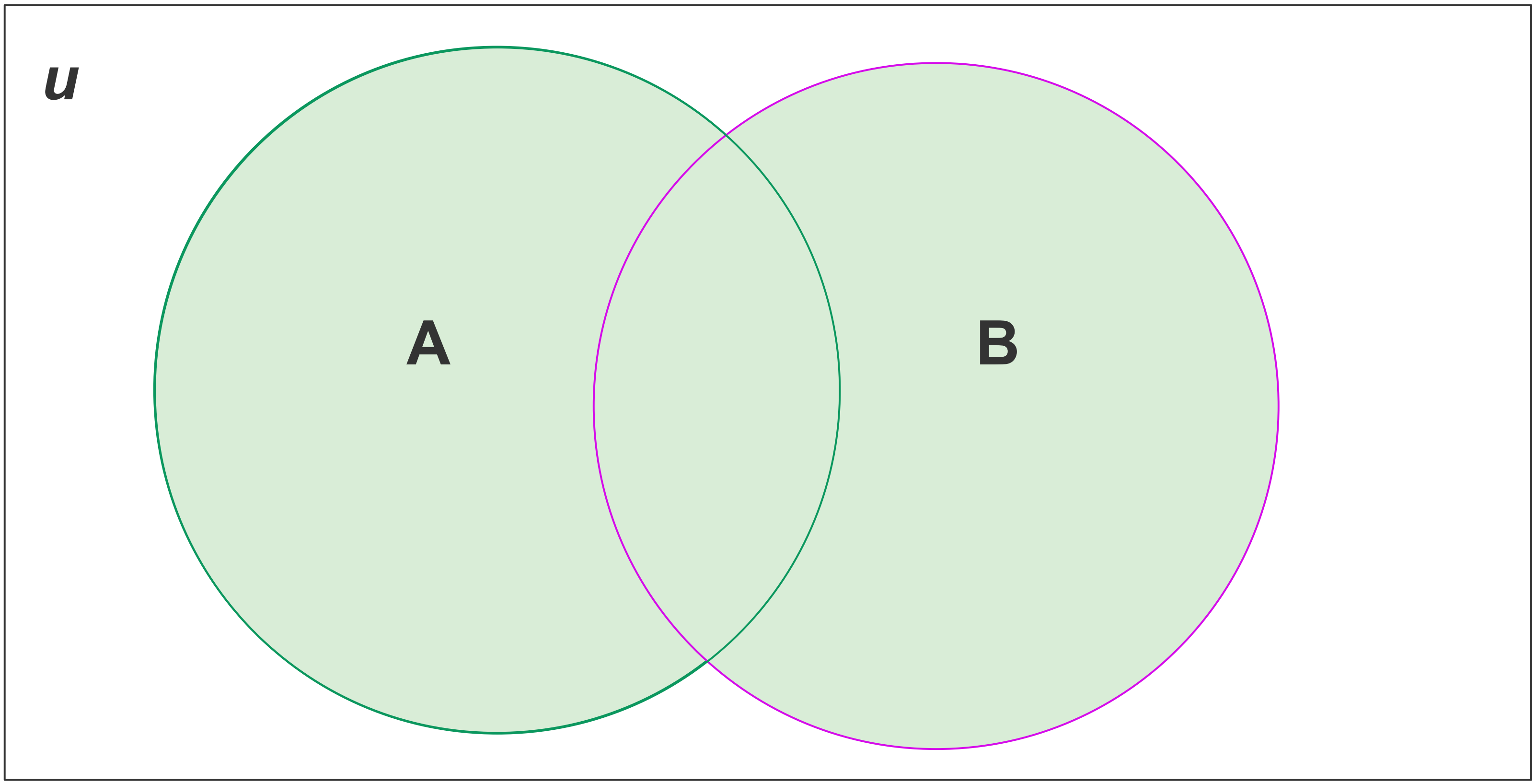
Understand Sets Visually!

What Set Operation is this?



Understand Sets Visually!

What Set Operation is this?
Union: A U B



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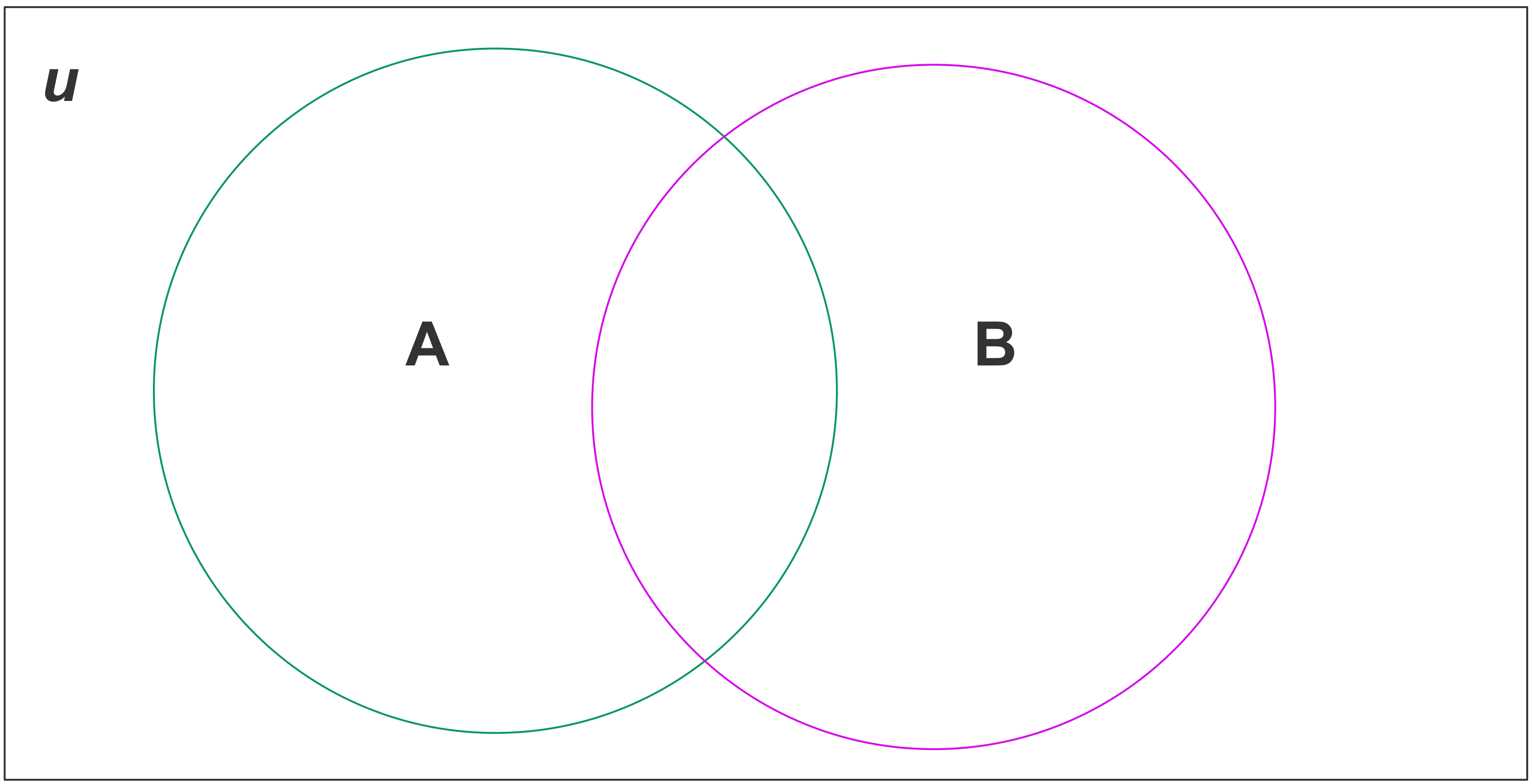

Understand Sets Visually!

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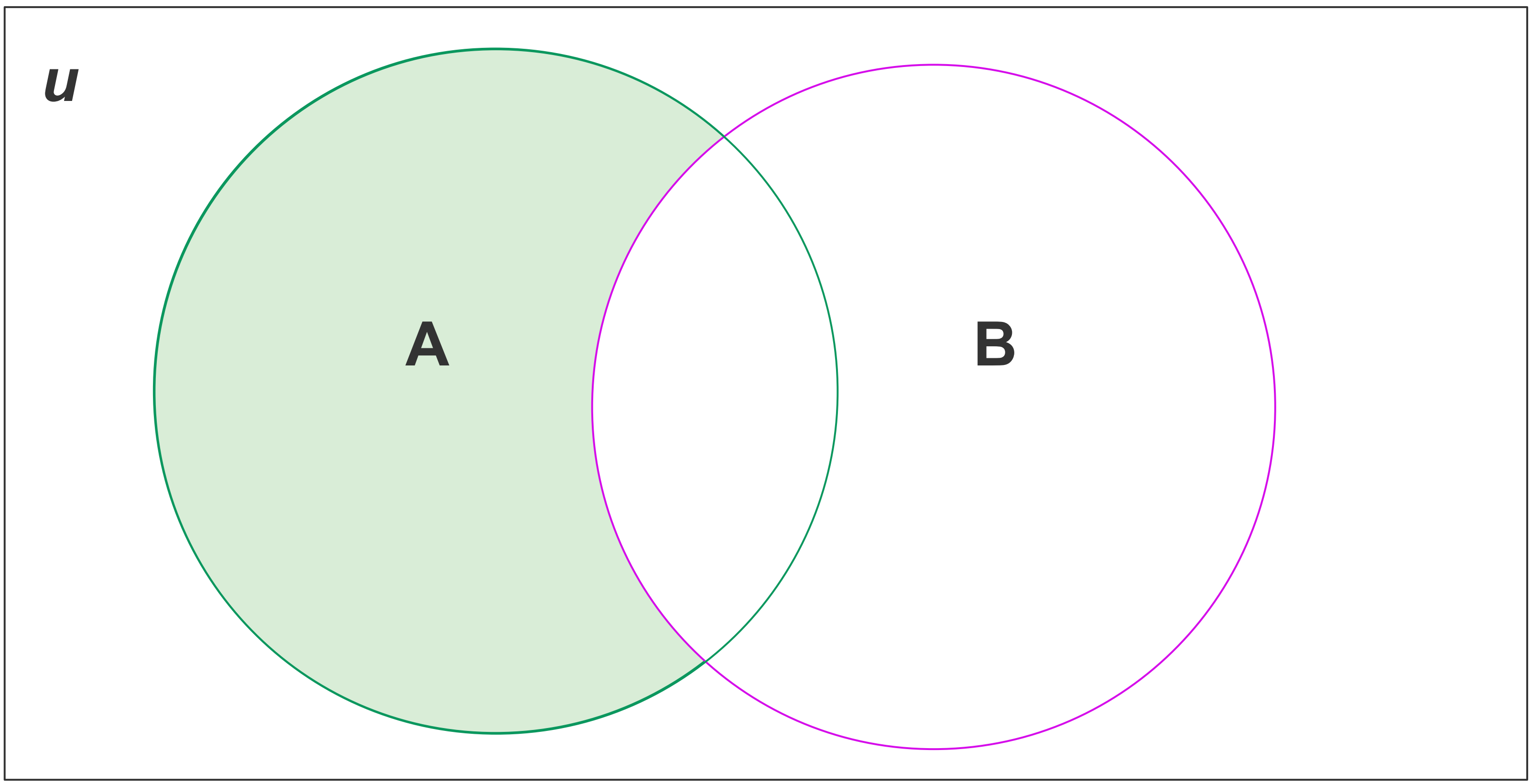
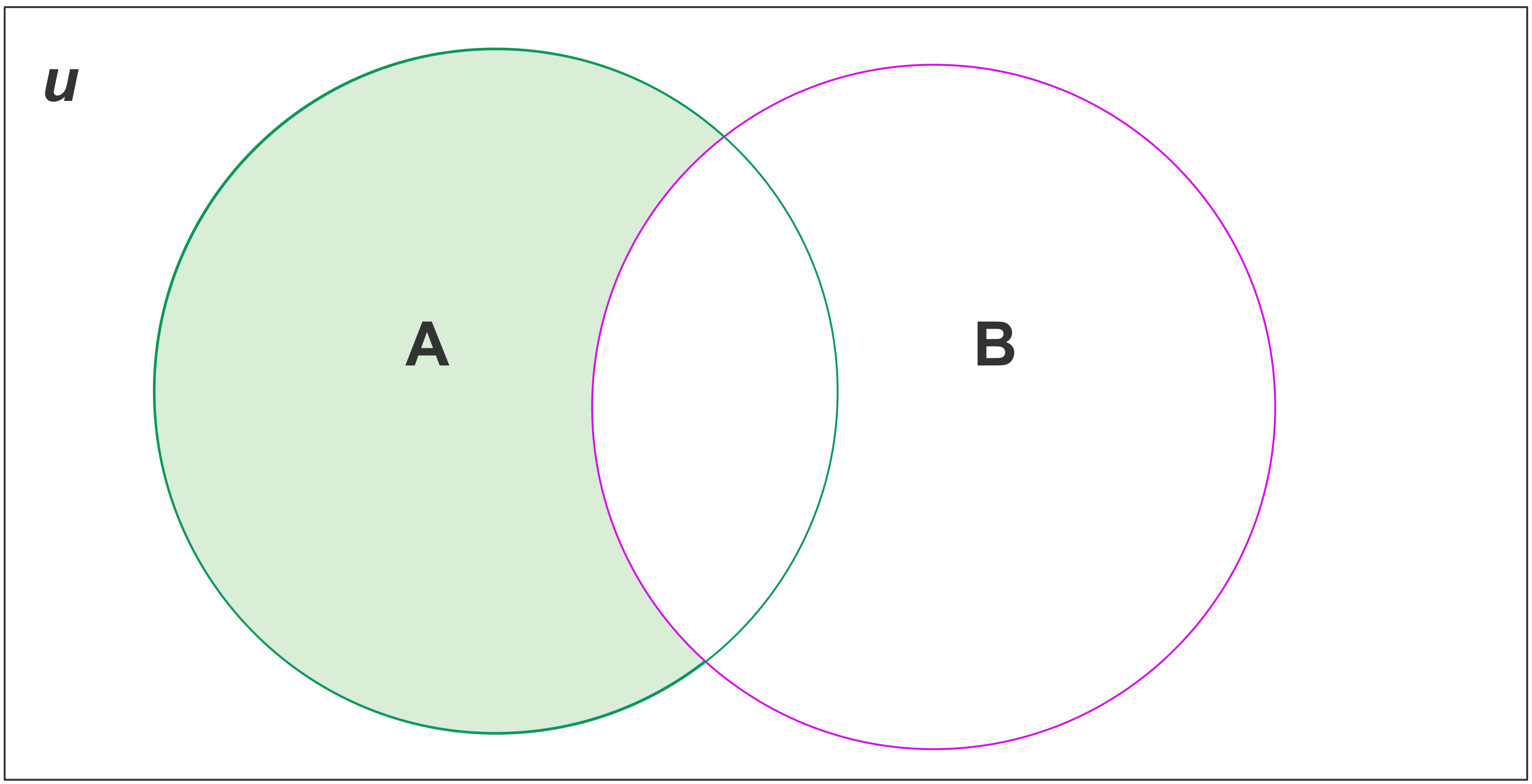
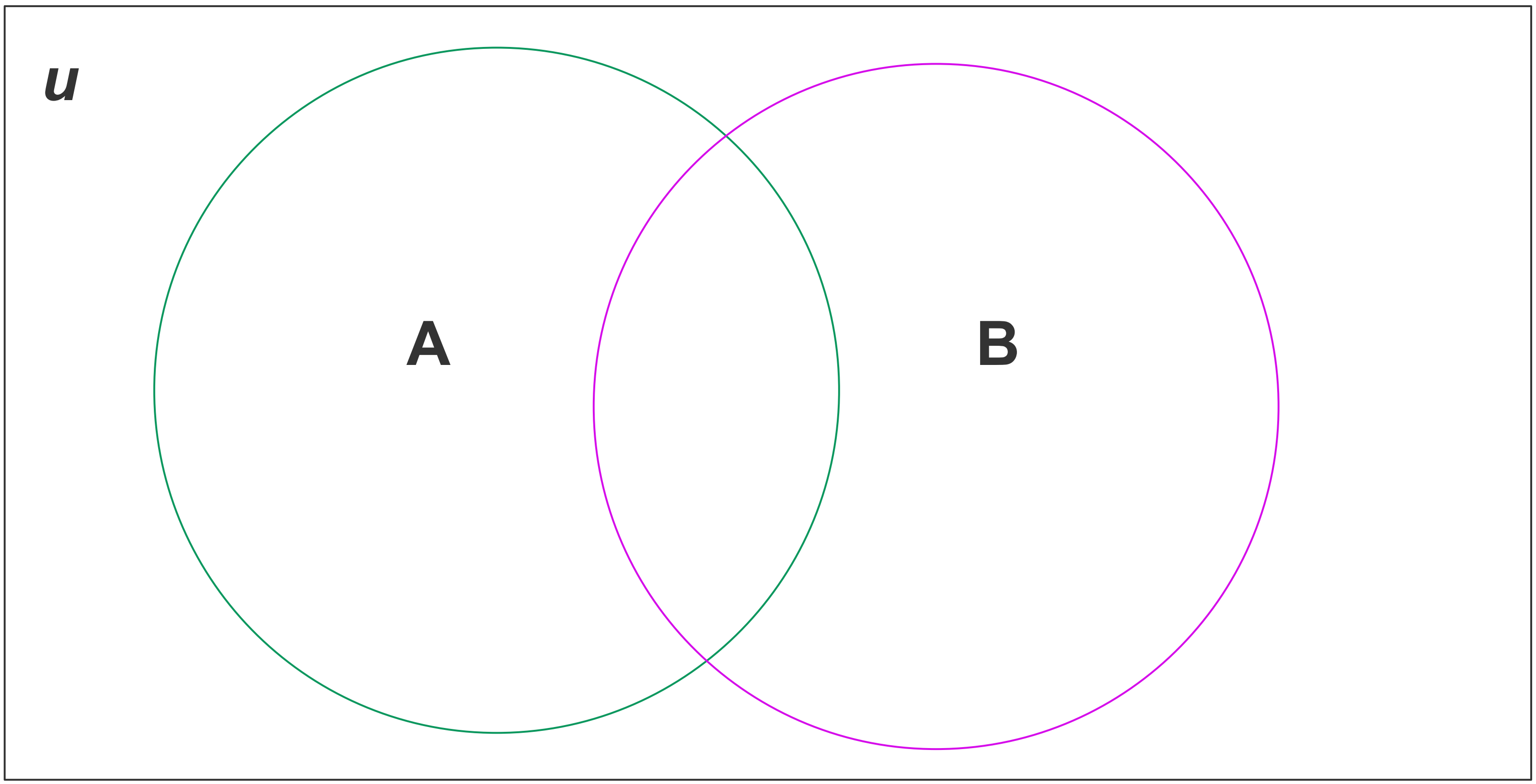


Understand Sets Visually!

What Set Operation is this?
Difference: A \ B

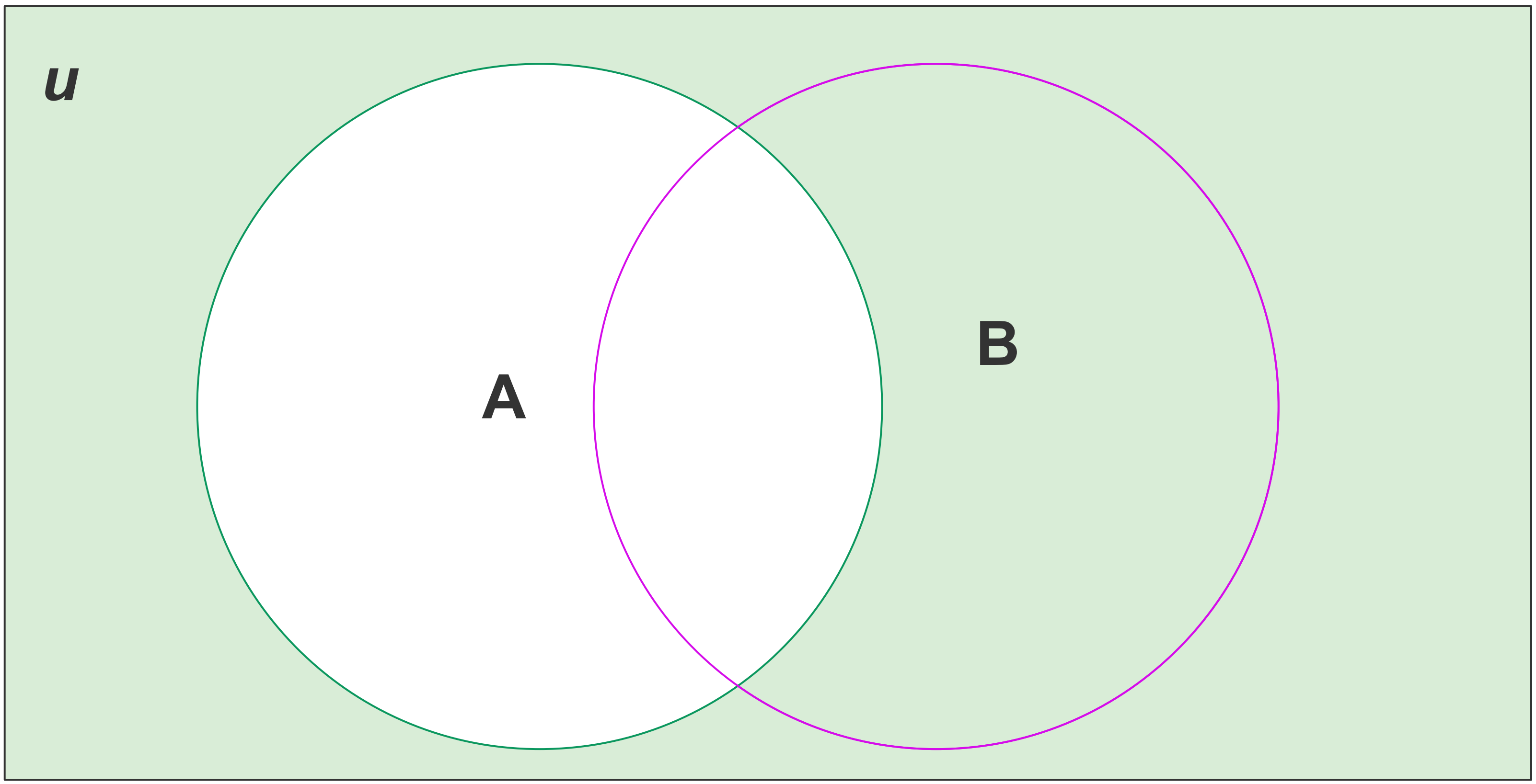


Understand Sets Visually!



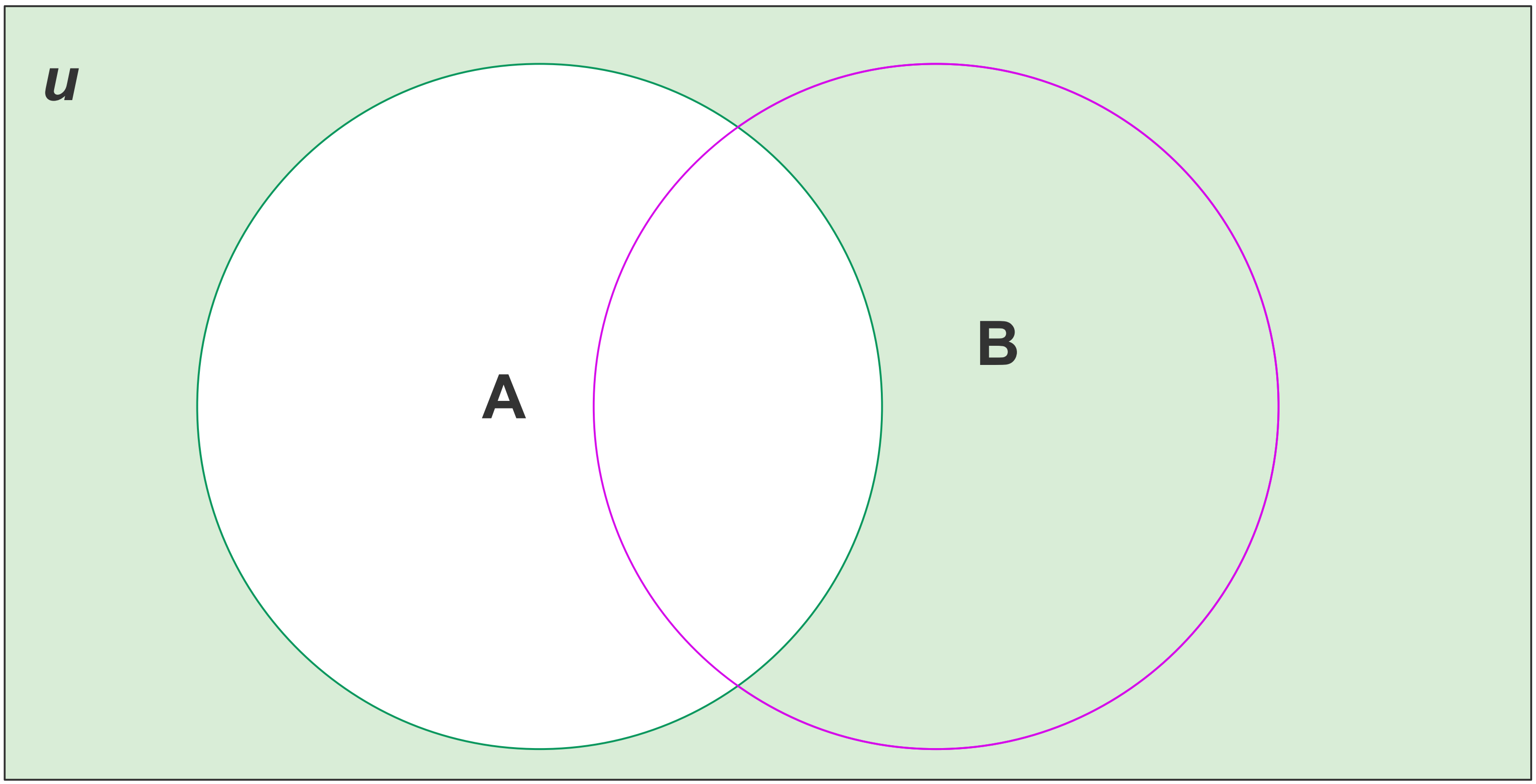
Understand Sets Visually!

What Set Operation is this?



Understand Sets Visually!

What Set Operation is this?
A complement: $\overline{A}$



Set Proofs

Subset Proofs

One of the most common types of proofs you will be asked to write involving sets is a subset proof. That is, you will be asked to prove that $A \subseteq B$. We always approach these proofs with the same proof skeleton:

Let x be an arbitrary element of A, so $x \in A$.
$\ldots$ some steps using set definitions to show that x must also be in B$\ldots$
Thus, $x \in B$
Since x was arbitrary, $A \subseteq B$.

Using Cozy For Sets

- **A U B**: A Union B- “A cup B”
- **A ∩ B**: “A cap B”
- **A ∊ B**: “A in B”
- A \ B: “A \ B”
- B complement- “~B” (<u>Only one Argument</u>)
- A\B\C is implicitly (A\B)\C

Problem 2a – Subsets

For any sets A, B, and C, show that it holds that $A \setminus B \subseteq A \cup C$

Set Equality: Using Meta Theorem

Problem 3

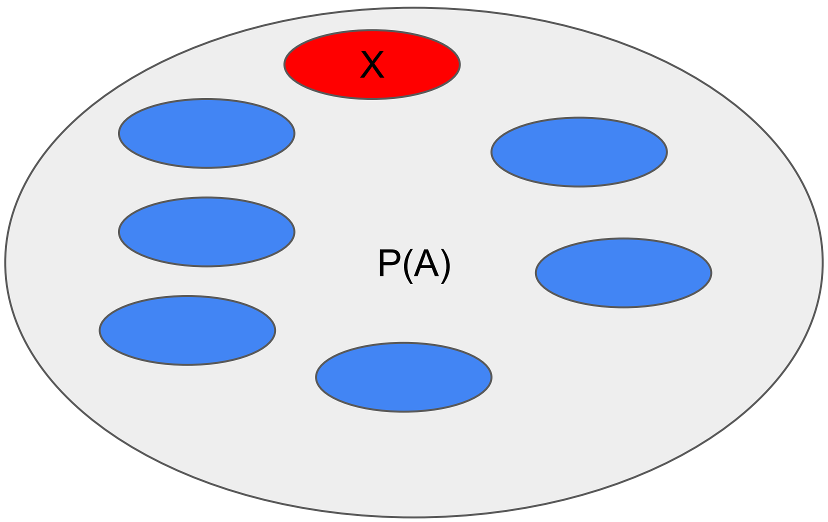
Let A and B be sets. Consider the claim: $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.

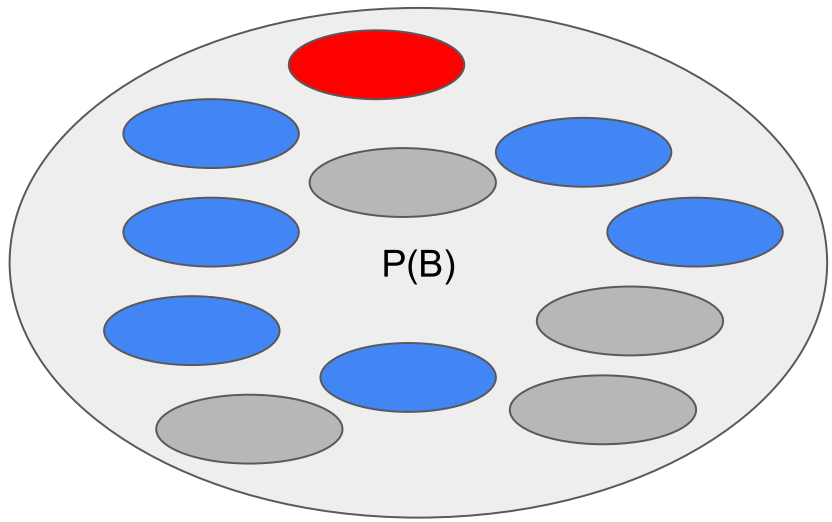
Powerset English Proof (optional)

Problem 4

Let A and B be sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ follows from $A \subseteq B$.

Powersets
P(A) subset P(B)





That’s all Folks!

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