Quiz Section 6: Midterm Review

Task 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- $\texttt{soy}(x)$ is true iff x contains soy milk.
- $\texttt{whole}(x)$ is true iff x contains whole milk.
- $\texttt{sugar}(x)$ is true iff x contains sugar
- $\texttt{decaf}(x)$ is true iff x is not caffeinated.
- $\texttt{vegan}(x)$ is true iff x is vegan.
- $\texttt{RobbieLikes}(x)$ is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like $=$ and $\neq$.

a) Coffee drinks with whole milk are not vegan.

b) Robbie only likes one coffee drink, and that drink is not vegan.

c) There is a drink that has both sugar and soy milk.

d) Translate the contrapositive of part (a) and write a matching (natural) English sentence.

e) Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\texttt{decaf}(x) \wedge \texttt{RobbieLikes}(x)] \rightarrow \texttt{sugar}(x))$$

f) Write the negation of part (e) in predicate logic and translate it into a (natural) English sentence. Take advantage of domain restriction.

Task 2 – Remains to be Seen

Prove the following for all integers $x, y \in \mathbb{Z}$:

$$\text{If } x \equiv_6 1 \text{ and } y \equiv_5 3 \text{ then } 5x + 3y \equiv_{15} 14.$$

a) Let your domain be integers. Write the predicate logic of this claim.

b) Write a formal proof for this claim.

Task 3 – Even, Odd, Even, Odd

Show that for any integers x, y, z, where $x + y$ is even and $y + z$ is odd that $x - z$ is odd.

a) Let your domain be integers. Write the predicate logic of this claim. Define predicates Odd and Even!

b) Write a formal proof for this claim.

Task 4 – 311 is Prime!

Find all solutions in the range $0 \leqslant x < 311$ to the modular equation:

$$12x \equiv 5 \pmod{311}$$

Task 5 – One small base case, one giant leap

Prove for all $n \in \mathbb{N}$ that the following identity is true:

$$\sum_{i=0}^{n} x^i = \frac{1 - x^{n+1}}{1 - x}$$

where $x \in \mathbb{R}, x \neq 1$.

Task 6 – Mathematical Dominoes

Prove that for every real number x and even integer r,

$$(1+x)^r \geqslant 1 + rx.$$

Hint: Use the definition of even to write $r = 2k$ and then induct on k.

Task 7 – Step one: Assume victory

Consider the following recursive function:

$$f(n) = \begin{cases} 4 & n = 2 \\ 12 & n = 3 \\ \frac{f(n-1)\cdot n}{2(n-2)} + \frac{f(n-2)\cdot n(n-1)}{2(n-3)(n-2)} & n \geqslant 4 \end{cases}$$

Prove $f(n) = 2n(n-1)$ for all integers $n \geqslant 2$.

You **must** use induction; be sure to define a predicate $P()$ and use good style.

Hint: Don't distribute (i.e., FOIL) terms before you have to in your inductive step. Look for things that might cancel.