CSE 311 Section MR

Midterm Review

Administrivia

Announcements & Reminders

- HW5
  - Was due yesterday @ 11:00 PM
  - Use late days if you need to!
  - Make sure you tagged pages on gradescope correctly

- Midterm is Coming Next Week!!!
  - Wednesday 2/19!
  - Book 1-on-1s on the ed message board!

Problem 1: Translation

Problem 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinated.
- vegan(x) is true iff x is vegan.
- RobbieLikes(x) is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like = and ≠.

a) Coffee drinks with whole milk are not vegan

b) Robbie only likes one coffee drink, and that drink is not vegan

c) There is a drink that has both sugar and soy milk.

Work on this problem with the people around you.

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a) Coffee drinks with whole milk are not vegan

a) Robbie only likes one coffee drink, and that drink is not vegan

a) There is a drink that has both sugar and soy milk.

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a) Coffee drinks with whole milk are not vegan

$$\forall x(\text{whole}(x) \rightarrow \neg\text{vegan}(x))$$

a) Robbie only likes one coffee drink, and that drink is not vegan

a) There is a drink that has both sugar and soy milk.

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(a) Coffee drinks with whole milk are not vegan.

(d) Translate the contrapositive of part (a) and write a matching (natural) English sentence.

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Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\text{decaf}(x) \land \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

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$$\forall x([\text{decaf}(x) \land \text{RobbieLikes}(x)] \to \text{sugar}(x))$$

Problem 2: Formal Proof

Problem 2 – Mod Proof

Prove the following for all integers $x, y \in \mathbb{Z}$:

$$\text{If } x \equiv_6 1 \text{ and } y \equiv_5 3 \text{ then } 5x + 3y \equiv_{15} 14.$$

a) Let your domain be integers. Write the predicate logic of this claim.

Problem 2 – Mod Proof

Prove the following for all integers $x, y \in \mathbb{Z}$:

$$\text{If } x \equiv_6 1 \text{ and } y \equiv_5 3 \text{ then } 5x + 3y \equiv_{15} 14.$$

a) Let your domain be integers. Write the predicate logic of this claim.

$$\forall x \forall y [(x \equiv_6 1) \land (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Problem 2 – Mod Proof

$\forall x \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$

b) Write a formal proof for this claim.

Problem 6: Induction

Problem 6 – Induction!

Prove that for every real number x and even integer r,

$$(1+x)^r \geqslant 1 + rx.$$

Since r is even, we can write $r = 2k$ for some integer $k \geqslant 0$. We will prove this by induction on k.

Define $P(k) :=$ "$(1+x)^{2k} \geqslant 1 + 2kx$" and prove using induction that $P(k)$ holds for all integers $k \geqslant 0$.

Problem 7: Strong Induction

Problem 7 – Induction!

Consider the following recursive function:

$$f(n) = \begin{cases} 4 & n = 2 \\ 12 & n = 3 \\ \frac{f(n-1) \cdot n}{2(n-2)} + \frac{f(n-2) \cdot n(n-1)}{2(n-3)(n-2)} & n \geqslant 4 \end{cases}$$

Prove $f(n) = 2n(n-1)$ for all integers $n \geqslant 2$.

Problem 7 – Induction!

$$f(n) = \begin{cases} 4 & n = 2 \\ 12 & n = 3 \\ \frac{f(n-1)\cdot n}{2(n-2)} + \frac{f(n-2)\cdot n(n-1)}{2(n-3)(n-2)} & n \geqslant 4 \end{cases}$$

Prove $f(n) = 2n(n-1)$ for all integers $n \geqslant 2$.

Let $P(n) :=$ "$f(n) = 2n(n-1)$". We prove $P(n)$ holds for all integers $n \geqslant 2$ using induction.