CSE 311 Section 3

Quantifiers and Proofs

Administrivia & Introductions

Announcements & Reminders

- HW1 out
  - If you think something was graded incorrectly, submit a regrade request!
  - Regrades generally will be open for a week

- HW2 was due yesterday 1/22 on Gradescope
  - Use a late day if you need to!
  - Gradescope: Make sure you select the pages for each question correctly
  - !! Selecting the pages after the deadline won't mark it as late

- HW3
  - Due Wednesday 1/29 @ 11:00pm

Announcements & Reminders

- Solidify your learning with 1 on 1 meetings!
  - Ask conceptual questions
  - Prep for exams
  - Previous homework
  - Walk through section problems
- These **are not** a time to talk about the <u>current homework</u> questions
  - We intend for office hours to be used for current assignments as we would not have time to give meetings to everyone if they were covered in the 1 on 1s.

Quantifiers

Problem 1 – Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other, if they are equivalent, or neither.

a) $\forall x\, \forall y\, P(x, y)$ $\quad$ $\forall y\, \forall x\, P(x, y)$

b) $\exists x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \exists x\, P(x, y)$

c) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\forall y\, \exists x\, P(x, y)$

d) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists x\, \forall y\, P(x, y)$

e) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \forall x\, P(x, y)$

Work on parts (d) and (e) with the people around you, and then we'll go over it together!

Problem 1 – Quantifier Switch

d) $\forall x \; \exists y \; P(x, y)$ $\qquad\qquad$ $\exists x \; \forall y \; P(x, y)$

Problem 1 – Quantifier Switch

d) $\forall x \, \exists y \, P(x, y)$ $\exists x \, \forall y \, P(x, y)$

Different!

For all x, there is a y **vs** there exists an x, that, for all y

Let P(x,y) be person x owns dog y

"All people own a dog"

	Robbie	Aruna	Anna	Jacob
				
				
				
				

VS

"There is person that owns all dogs"

	Robbie	Aruna	Anna	Jacob
				
				
				
				

Problem 1 – Quantifier Switch

d) $\forall x\, \exists y\, P(x,y)$ $\qquad$ $\exists x\, \forall y\, P(x,y)$

Different!

For all x, there is a y **vs** there exists an x, that, for all y

Let P(x,y) be person x owns dog y

"All people own a dog"

	Robbie	Aruna	Anna	Jacob
	X			
				X
		X		
			X	

VS

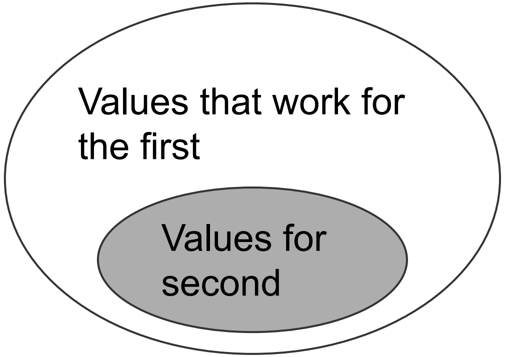
"There is person that owns all dogs"

	Robbie	Aruna	Anna	Jacob
	X			
	X			
	X			
	X			

Problem 1 – Quantifier Switch

e) $\forall x \, \exists y \, P(x, y)$ $\qquad$ $\exists y \, \forall x \, P(x, y)$

Problem 1 – Quantifier Switch



e) $\forall x\, \exists y\, P(x, y)$ $\qquad\qquad$ $\exists y\, \forall x\, P(x, y)$

The second implies the first

For all x, there is a y **vs** there exists a y, that, for all x

The second is **stronger** since a **specific y** must work **for all x** whereas for the first, the y value **does not** have to be the same **for every x**

"All people own a dog"

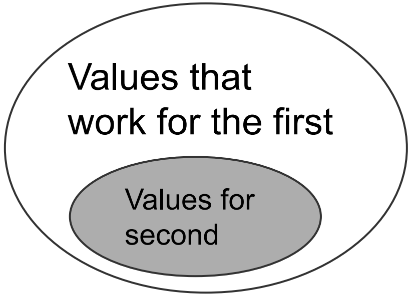
	Robbie	Aruna	Anna	Jacob
				
				
				
				

VS

"There is a dog owned by all people"

	Robbie	Aruna	Anna	Jacob
				
				
				
				

Problem 1 – Quantifier Switch



e) $\forall x \, \exists y \; P(x, y)$ $\qquad\qquad$ $\exists y \, \forall x \; P(x, y)$

The second implies the first

For all x, there is a y **vs** there exists a y, that, for all x

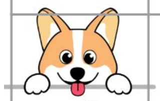
The second is **stronger** since a **specific y** must work **for all x** whereas for the first, the y value **does not** have to be the same **for every x**

"All people own a dog"

	Robbie	Aruna	Anna	Jacob
	X			
				X
		X		
			X	

VS

"There is a dog owned by all people"

	Robbie	Aruna	Anna	Jacob
				
				
	X	X	X	X
				

Find the Bug!

Problem 3 (a,b)- Find the Bug

Each of these inference proofs is incorrect. Identify the line (or lines) which incorrectly apply a law, and explain the error. Then, if the claim is false, give concrete examples of propositions to show it is false. If it is true, write a correct proof.

This proof claims to show that given $a \to (b \vee c)$, we can conclude $a \to c$.

1. $a \to (b \vee c)$ [Given]
   - 2.1. a [Assumption]
   - 2.2. $\neg b$ [Assumption]
   - 2.3. $b \vee c$ [Modus Ponens, from 1 and 2.1]
   - 2.4. c [$\vee$ elimination, from 2.2 and 2.3]
2. $a \to c$ [Direct Proof Rule, from 2.1-2.4]

Problem 3 (a,b)- Find the Bug

This proof claims to show that given $a \to (b \vee c)$, we can conclude $a \to c$.

1.	$a \to (b \vee c)$	[Given]
2.1.	a	[Assumption]
2.2.	$\neg b$	[Assumption]
2.3.	$b \vee c$	[Modus Ponens, from 1 and 2.1]
2.4.	c	[$\vee$ elimination, from 2.2 and 2.3]
2.	$a \to c$	[Direct Proof Rule, from 2.1-2.4]

Cannot use **two** assumptions for direct proof

Problem 3 (a,b)- Find the Bug

This proof claims to show that given $a \to (b \vee c)$, we can conclude $a \to c$.

1. $a \to (b \vee c)$ [Given]
   - 2.1. a [Assumption]
   - 2.2. $\neg b$ [Assumption]
   - 2.3. $b \vee c$ [Modus Ponens, from 1 and 2.1]
   - 2.4. c [$\vee$ elimination, from 2.2 and 2.3]
2. $a \to c$ [Direct Proof Rule, from 2.1-2.4]

Conclusion does not make sense! We cannot conclude c from a like this

Problem 3 (a,b)- Find the Bug

1. $p \rightarrow q$ [Given]
2. r [Given]
3. $p \rightarrow (q \vee r)$ [Intro $\vee$ (1,2)]

Problem 3 (a,b)- Find the Bug

1. $p \to q$ [Given]
2. r [Given]
3. $p \to (q \vee r)$ [Intro $\vee$ (1,2)]

We're applying the rule to only a subexpression! To fix this:

Problem 3 (a,b)- Find the Bug

1. $p \rightarrow q$ [Given]
2. r [Given]
3. $p \rightarrow (q \vee r)$ [Intro $\vee$ (1,2)]

We're applying the rule to only a subexpression! To fix this:
Take an assumption p and use direct proof rule

Formal Proof + Cozy

Problem 4- Formal Proof

Show that $\neg t \rightarrow s$ follows from $t \vee q$ and $q \rightarrow r$ and $r \rightarrow s$ with a formal proof.

Problem 4- Formal Proof

Show that $\neg t \to s$ follows from $t \vee q$ and $q \to r$ and $r \to s$ with a formal proof.

Problem 4- Formal Proof

Show that $\neg t \to s$ follows from $t \vee q$ and $q \to r$ and $r \to s$ with a formal proof.

Formal proof:

1.	$t \vee q$	[Given]
2.	$q \to r$	[Given]
3.	$r \to s$	[Given]

Problem 4- Formal Proof

Show that $\neg t \to s$ follows from $t \vee q$ and $q \to r$ and $r \to s$ with a formal proof.

Formal proof:

1. $t \vee q$ [Given]
2. $q \to r$ [Given]
3. $r \to s$ [Given]

    4.1. $\neg t$ [Assumption]

4. $\neg t \to s$ [Direct Proof Rule]

Problem 4- Formal Proof

Show that ¬t → s follows from t v q and q → r and r → s with a formal proof.

Formal proof:

1. $t \vee q$ [Given]
2. $q \rightarrow r$ [Given]
3. $r \rightarrow s$ [Given]
    - 4.1. $\neg t$ [Assumption]
    - 4.2. q [Elim of $\vee$: 1, 4.1]
4. $\neg t \rightarrow s$ [Direct Proof Rule]

Problem 4- Formal Proof

Show that $\neg t \to s$ follows from $t \vee q$ and $q \to r$ and $r \to s$ with a formal proof.

Formal proof:

1.	$t \vee q$		[Given]
2.	$q \to r$		[Given]
3.	$r \to s$		[Given]
	4.1.	$\neg t$	[Assumption]
	4.2.	q	[Elim of $\vee$: 1, 4.1]
	4.3.	r	[MP of 4.2, 2]
4.	$\neg t \to s$		[Direct Proof Rule]

Problem 4- Formal Proof

Show that $\neg t \to s$ follows from $t \vee q$ and $q \to r$ and $r \to s$ with a formal proof.

Formal proof:

1.	$t \vee q$		[Given]
2.	$q \to r$		[Given]
3.	$r \to s$		[Given]
	4.1.	$\neg t$	[Assumption]
	4.2.	q	[Elim of $\vee$: 1, 4.1]
	4.3.	r	[MP of 4.2, 2]
	4.4.	s	[MP 4.3, 3]
4.	$\neg t \to s$		[Direct Proof Rule]

Problem 4- Formal Proof

Show that $\neg t \rightarrow s$ follows from $t \vee q$ and $q \rightarrow r$ and $r \rightarrow s$ with a formal proof.

Now that you attempted this on paper, we can check on Cozy:

tinyurl.com/cse311-s3-4

Formal Proof + Cozy [Extra]

Problem 5- Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$ with a formal proof. Then, translate your proof to English.

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]

Problem 5- Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$ with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]

Problem 5- Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$ with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \rightarrow s$	[Elim of $\wedge$: 3]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \to s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \to s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ⋀ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \to s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \to s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]
11.	$\neg q$	[Elim of $\vee$: 10, 9]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \to s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]
11.	$\neg q$	[Elim of $\vee$: 10, 9]
12.	$p \to q$	[Elim of $\wedge$: 3]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \rightarrow s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]
11.	$\neg q$	[Elim of $\vee$: 10, 9]
12.	$p \rightarrow q$	[Elim of $\wedge$: 3]
13.	$\neg q \rightarrow \neg p$	[Contrapositive: 12]

Problem 5- Formal Proof

Show that ¬p follows from ¬(¬r v t), ¬q v ¬s and (p→q) ∧ (r→s) with a formal proof.

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \rightarrow s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]
11.	$\neg q$	[Elim of $\vee$: 10, 9]
12.	$p \rightarrow q$	[Elim of $\wedge$: 3]
13.	$\neg q \rightarrow \neg p$	[Contrapositive: 12]
14.	$\neg p$	[MP: 11,13]

Problem 5- Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$ with a formal proof. Then, translate your proof to English.

Now that you have tried this on paper, you can attempt it here:
tinyurl.com/cse311-s3-5

Fill in Formal Logic (extra)

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x\,(T(x) \rightarrow M(x))$ ____________________

    2.1. $\exists x\, T(x)$ ____________________

    2.2. $T(c)$ ____________________

    2.3. $T(c) \rightarrow M(c)$ ____________________

    2.4. $M(c)$ ____________________

    2.5. $\exists y\, M(y)$ ____________________

2. $(\exists x\, T(x)) \rightarrow (\exists y\, M(y))$ ____________________

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x\,(T(x) \to M(x))$ Given ______

    2.1. $\exists x\, T(x)$ ______

    2.2. $T(c)$ ______

    2.3. $T(c) \to M(c)$ ______

    2.4. $M(c)$ ______

    2.5. $\exists y\, M(y)$ ______

2. $(\exists x\, T(x)) \to (\exists y\, M(y))$ ______

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x \, (T(x) \rightarrow M(x))$ Given ______

    2.1. $\exists x \, T(x)$ Assumption ______

    2.2. $T(c)$ ______

    2.3. $T(c) \rightarrow M(c)$ ______

    2.4. $M(c)$ ______

    2.5. $\exists y \, M(y)$ ______

2. $(\exists x \, T(x)) \rightarrow (\exists y \, M(y))$ ______

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x\,(T(x) \to M(x))$	Given
2.1. $\exists x\, T(x)$	Assumption
2.2. $T(c)$	Elim ∃ : 2.1 (c)
2.3. $T(c) \to M(c)$	
2.4. $M(c)$	
2.5. $\exists y\, M(y)$	
2. $(\exists x\, T(x)) \to (\exists y\, M(y))$	

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1.	$\forall x\,(T(x) \to M(x))$	Given
2.1.	$\exists x\, T(x)$	Assumption
2.2.	$T(c)$	Elim ∃: 2.1 (c)
2.3.	$T(c) \to M(c)$	Elim ∀: 1
2.4.	$M(c)$	
2.5.	$\exists y\, M(y)$	
2.	$(\exists x\, T(x)) \to (\exists y\, M(y))$	

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1.	$\forall x\,(T(x) \to M(x))$	Given
2.1.	$\exists x\, T(x)$	Assumption
2.2.	$T(c)$	Elim ∃: 2.1 (c)
2.3.	$T(c) \to M(c)$	Elim ∀: 1
2.4.	$M(c)$	Modus Ponens: 2.2, 2.3
2.5.	$\exists y\, M(y)$	
2.	$(\exists x\, T(x)) \to (\exists y\, M(y))$	

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x\,(T(x) \to M(x))$	Given
2.1. $\exists x\, T(x)$	Assumption
2.2. $T(c)$	Elim ∃: 2.1 (c)
2.3. $T(c) \to M(c)$	Elim ∀: 1
2.4. $M(c)$	Modus Ponens: 2.2, 2.3
2.5. $\exists y\, M(y)$	Intro ∃: 2.4
2. $(\exists x\, T(x)) \to (\exists y\, M(y))$	

Problem 8- Predicate Logic Formal Proof

Given ∀(T(x) → M(x)) , we wish to prove (∃ T(x)) →(∃ M(y)). The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof. Then, translate the proof to English.

1. $\forall x\,(T(x) \rightarrow M(x))$	Given
2.1. $\exists x\, T(x)$	Assumption
2.2. $T(c)$	Elim ∃: 2.1 (c)
2.3. $T(c) \rightarrow M(c)$	Elim ∀: 1
2.4. $M(c)$	Modus Ponens: 2.2, 2.3
2.5. $\exists y\, M(y)$	Intro ∃: 2.4
2. $(\exists x\, T(x)) \rightarrow (\exists y\, M(y))$	Direct Proof: 2.1-2.5

That's All, Folks!

Thanks for coming to section this week!
Any questions?