CSE 311 Section 2

Logic and Equivalences

Administrivia & Introductions

Announcements & Reminders

- Sections are Graded
    - You will be graded on section participation, so please try to come ☺
    - If you cannot attend you will need to submit ALL the section problems to gradescope

- HW1 due YESTERDAY **(1/15) @ 11:00 PM** on Gradescope
    - Homework is usually due **Wednesdays @ 11:00pm**, released Thursday evening
    - Remember, you only have 3 late days to use throughout the quarter
    - You can use only 1 late days on any 1 assignment

- Check the course website for OH times!

- Concept Checks!
    - Absolute deadline on **Thursdays @ 11:59 pm**

Task 1: Predicate Logic

Translate these system specifications into English where $F(p)$ is “Printer p is out of service”, $B(p)$ is “Printer p is busy”, $L(j)$ is “Print job j is lost,” and $Q(j)$ is “Print job j is queued”. Let the domain be all printers and all print jobs.

a) $\exists p\,(F(p) \wedge B(p)) \rightarrow \exists j\; L(j)$

b) $(\forall j\; B(j)) \rightarrow (\exists p\, Q(p))$

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers and all print jobs.

a) $\exists p\,(F(p) \wedge B(p)) \rightarrow \exists j\, L(j)$

If at least one printer is busy and out of service, then at least one job is lost.

b) $(\forall j\; B(j)) \rightarrow (\exists p\; Q(p))$

If all printers are busy, then there is a queued job.

Tautology if it is always true
Contradiction if it is always false
Contingency if it can be either true or false

Task 2: Symbolic Proofs with Cozy

Task 2 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \lor r)$

Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

p	q	r	$\neg p$	$(q \rightarrow r)$	$(p \vee r)$	$\neg p \rightarrow (q \rightarrow r)$	$q \rightarrow (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Remember these identities!

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \rightarrow q \equiv \neg p \vee q$
- Contrapositive
  - $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv$	$\neg\neg p \vee (q \rightarrow r)$	Law of Implication

Task 2 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$$
\begin{aligned}
\neg p \to (q \to r) &\equiv \neg\neg p \vee (q \to r) && \text{Law of Implication} \\
&\equiv p \vee (q \to r) && \text{Double Negation}
\end{aligned}
$$

Task 2 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication

Task 2 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \lor r)$

$$
\begin{aligned}
\neg p \to (q \to r) \quad & \equiv \quad \neg\neg p \lor (q \to r) && \text{Law of Implication} \\
& \equiv \quad p \lor (q \to r) && \text{Double Negation} \\
& \equiv \quad p \lor (\neg q \lor r) && \text{Law of Implication} \\
& \equiv \quad (p \lor \neg q) \lor r && \text{Associativity}
\end{aligned}
$$

Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv$	$\neg\neg p \vee (q \rightarrow r)$	Law of Implication
	$\equiv$	$p \vee (q \rightarrow r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$(\neg q \vee p) \vee r$	Commutativity

Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv$	$\neg\neg p \vee (q \rightarrow r)$	Law of Implication
	$\equiv$	$p \vee (q \rightarrow r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$(\neg q \vee p) \vee r$	Commutativity
	$\equiv$	$\neg q \vee (p \vee r)$	Associativity

Task 2 – Equivalences

b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv$	$\neg\neg p \vee (q \to r)$	Law of Implication
	$\equiv$	$p \vee (q \to r)$	Double Negation
	$\equiv$	$p \vee (\neg q \vee r)$	Law of Implication
	$\equiv$	$(p \vee \neg q) \vee r$	Associativity
	$\equiv$	$(\neg q \vee p) \vee r$	Commutativity
	$\equiv$	$\neg q \vee (p \vee r)$	Associativity
	$\equiv$	$q \to (p \vee r)$	Law of Implication

COZY: Task 2 – Equivalences

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

tinyurl.com/CSE311S21b

Remember these identities!

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
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- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \rightarrow q \equiv \neg p \vee q$
- Contrapositive
  - $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Task 4: Circuits

Task 4a - Circuits

a) Let Q be defined by $Q(p, q) = (\neg p) \oplus q$. Using only NOT, OR and Q gates express the logical expression $(a \wedge b) \oplus c$.

Task 4a - Circuits

a) Let Q be defined by $Q(p, q) = (\neg p) \oplus q$. Using only NOT, OR and Q gates express the logical expression $(a \wedge b) \oplus c$.

$(a \wedge b) \oplus c$	$\equiv$	$\neg\neg(a \wedge b) \oplus c$	Double Negation
	$\equiv$	$\neg(\neg a \vee \neg b) \oplus c$	De Morgan
	$\equiv$	$Q((\neg a \vee \neg b), c)$	Definition of Q

Task 6: CNF and Simplification

Task 6b – Canonical Forms

b) Write the **CNF** expressions for G(A,B,C)

A	*B*	*C*	*G(A,B,C)*
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

Work on part (b) with the people around you, and then we'll go over it together!

Task 6b – Canonical Forms

b) Write the **CNF** expressions for G(A,B,C)

A	B	C	G(A,B,C)	
1	1	1	1	
1	1	0	1	
1	0	1	1	
1	0	0	0	(A'+B+C')
0	1	1	1	
0	1	0	0	(A+B'+C)
0	0	1	1	
0	0	0	0	(A+B+C)

(A'+B+C)(A+B'+C)(A+B+C)

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

Identity
$A \wedge \mathsf{T} \equiv A$
$A \vee \mathsf{F} \equiv A$

Domination
$A \vee \mathsf{T} \equiv \mathsf{T}$
$A \wedge \mathsf{F} \equiv \mathsf{F}$

Idempotency
$A \vee A \equiv A$
$A \wedge A \equiv A$

Commutativity
$A \vee B \equiv B \vee A$
$A \wedge B \equiv B \wedge A$

Associativity
$(A \vee B) \vee C \equiv A \vee (B \vee C)$
$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$

Distributivity
$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

Absorption
$A \vee (A \wedge B) \equiv A$
$A \wedge (A \vee B) \equiv A$

Negation
$A \vee \neg A \equiv \mathsf{T}$
$A \wedge \neg A \equiv \mathsf{F}$

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$$ Idempotency

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C) \quad \text{Idempotency}$$

$$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C) \quad \text{Commutativity x 2}$$

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C) \quad \text{Idempotency}$$

$$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C) \quad \text{Commutativity x 2}$$

$$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B) \quad \text{Commutativity x 2}$$

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C) \quad \text{Idempotency}$$

$$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C) \quad \text{Commutativity x 2}$$

$$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B) \quad \text{Commutativity x 2}$$

$$= (B + C + A')(B + C + A)(A + C + B')(A + C + B) \quad \text{Commutativity x 2}$$

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$	Idempotency
$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C)$	Commutativity x 2
$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B)$	Commutativity x 2
$= (B + C + A')(B + C + A)(A + C + B')(A + C + B)$	Commutativity x 2
$= [(B + C) + (A' \cdot A)][(A + C) + (B' \cdot B)]$	Distributivity x2

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$	Idempotency
$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C)$	Commutativity x 2
$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B)$	Commutativity x 2
$= (B + C + A')(B + C + A)(A + C + B')(A + C + B)$	Commutativity x 2
$= [(B + C) + (A' \cdot A)][(A + C) + (B' \cdot B)]$	Distributivity x2
$= [(B + C) + 0][(A + C) + 0]$	Negation x 2

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$	Idempotency
$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C)$	Commutativity x 2
$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B)$	Commutativity x 2
$= (B + C + A')(B + C + A)(A + C + B')(A + C + B)$	Commutativity x 2
$= [(B + C) + (A' \cdot A)][(A + C) + (B' \cdot B)]$	Distributivity x2
$= [(B + C) + 0][(A + C) + 0]$	Negation x 2
$= [B + C][A + C]$	Identity

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$	Idempotency
$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C)$	Commutativity x 2
$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B)$	Commutativity x 2
$= (B + C + A')(B + C + A)(A + C + B')(A + C + B)$	Commutativity x 2
$= [(B + C) + (A' \cdot A)][(A + C) + (B' \cdot B)]$	Distributivity x2
$= [(B + C) + 0][(A + C) + 0]$	Negation x 2
$= [B + C][A + C]$	Identity
$= [C + B][C + A]$	Commutativity

Task 6c – Canonical Forms

$$(A' + B + C)(A + B' + C)(A + B + C)$$

$= (A' + B + C)(A + B' + C)(A + B + C)(A + B + C)$	Idempotency
$= (A' + B + C)(A + B + C)(A + B' + C)(A + B + C)$	Commutativity x 2
$= (A' + B + C)(A + B + C)(A + C + B')(A + C + B)$	Commutativity x 2
$= (B + C + A')(B + C + A)(A + C + B')(A + C + B)$	Commutativity x 2
$= [(B + C) + (A' \cdot A)][(A + C) + (B' \cdot B)]$	Distributivity x2
$= [(B + C) + 0][(A + C) + 0]$	Negation x 2
$= [B + C][A + C]$	Identity
$= [C + B][C + A]$	Commutativity
$= C + B \cdot A$	Distributivity

That’s All, Folks!

Thanks for coming to section this week!
Any questions?