# Section 03: Proof Techniques

## 1. Direct Proof

(a) Let the domain of discourse be integers. Define the predicates $Odd(x) := \exists k(x = 2k + 1)$, and $Even(x) := \exists k(x = 2k)$. Translate the following claim to predicate logic:

The sum of an even and odd integer is odd.

(b) Prove that the claim holds.

## 2. Proof of Biconditional

(a) Let the domain of discourse be integers. Define the predicates $Odd(x) := \exists k(x = 2k + 1)$, and $Even(x) := \exists k(x = 2k)$. Translate the following claim to predicate logic:

For all integers $n$, $n - 4$ is even if and only if $n + 17$ is odd.

(b) For each direction, write the first few sentences and last few sentences of the English proof.

(c) Prove that the claim holds.

## 3. Proof by Contrapositive

(a) Let the domain of discourse be integers. Define the predicates $Odd(x) := \exists k(x = 2k + 1)$ and $Even(x) := \exists k(x = 2k)$. Translate the following claim to predicate logic:

For all integers $x$, if $7x + 9$ is even, the $x$ is odd.

(b) Try to prove the claim directly. Do you get stuck?
Note that it is actually possible to write a direct proof, though it is slightly more difficult to see how.

(c) What is the contrapositive of the claim in predicate logic?

(d) Prove that the claim holds by proving the contrapositive.

## 4. Proof by Cases

Prove by cases that for all integers $n$, $n^2 - 3n$ is even.

## 5. Disproving a For All Claim

Disprove the following claim:

For all integers $a, b, c$ if $ac = bc$ then $a = b$.

## 6. Disproving a There Exists Claim

Consider the following claim:

There exists an integer $x$ such that $x$ is even and $x^2$ is odd.

(a) This claim is false. Without using any formal reasoning, what does your intuition say about how to disprove this claim?

(b) Let the domain of discourse be integers. Define the predicates $Odd(x) := \exists k(x = 2k + 1)$ and $Even(x) = \exists k(x = 2k)$. Translate the above claim to predicate logic.

(c) Negate the predicate logic translation. Then use a chain of logical equivalences to show that your negation is equivalent to $\forall x(Even(x) \rightarrow Even(x^2))$.

**Hint:** You may use the fact that $\neg Odd(a) \equiv Even(a)$.

(d) Recall that to disprove a claim, we must prove its negation. Part (c) shows us that to disprove the above claim, we should prove that if an integer $x$ is even, then $x^2$ is also even. Does this match your intuition?

(e) Write a proof of the fact that if an integer $x$ is even, then $x^2$ is also even.

(f) Celebrate! You have successfully disproved the claim!