

# Section 01: Solutions

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## 1. Warm-Up

Translate the English sentences below into symbolic logic.

- (a) If I am lifting weights this afternoon, then I do a warm-up exercise.

**Solution:**

Since we're in "if...then..." form, the sentence is an implication.

$p$ : I am lifting weights

$q$ : I do a warm-up exercise

$$p \rightarrow q$$

- (b) If I am cold and going to bed or I am two-years old, then I carry a blanket. **Solution:**

$p$ : I am cold

$q$ : I am going to bed

$r$ : I am two-years old

$s$ : I carry a blanket

$$[(p \wedge q) \vee r] \rightarrow s$$

How did we know the translation wasn't  $[p \wedge (q \vee r)] \rightarrow s$ ? Two hints were available: first, omitted words ("going to bed" instead of "I am going to bed" indicates  $q$  should be closer to the "and" than the "or"), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

## 2. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

- (a) whenever I walk my dog, I make new friends. **Solution:**

$p$ : I walk my dog

$q$ : I make new friends

$$p \rightarrow q$$

The promise is that we will definitely make new friends on the condition of walking our dog.

- (b) I will drink coffee, if Starbucks is open or my coffeemaker works.

**Solution:**

$p$ : I will drink coffee

$q$ : Starbucks is open

$r$ : my coffeemaker works

$$(q \vee r) \rightarrow p$$

- (c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote. **Solution:**

$p$ : One is a U.S. citizen  $q$ : One is over 18  $r$ : One is eligible to vote

$$(p \wedge q) \rightarrow r$$

The original sentence omits a subject. We introduced a dummy subject “one” to the propositions, you might have said “someone” or “a person” instead (among other options).

**Note:** You may wonder, doesn’t being eligible to vote imply you are a U.S. citizen and over 18 years old? In this case, yes, it does! However, this is not the question we are asking you to translate; this is something called the “contrapositive” of the original statement.

- (d) I can go home only if I have finished my homework. **Solution:**

$p$ : I can go home.  $q$ : I have finished my homework.

$$p \rightarrow q$$

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

“Only if” is one of the more confusing arrangements – the consequence (“the then part”) is adjacent to the “only if.”

- (e) Having an internet connection is necessary to log onto zoom. **Solution:**

$p$ : One has an internet connection

$q$ : One can log onto zoom

$$q \rightarrow p$$

The internet connection is not enough (what if you don’t have the meeting link?) but certainly if you are in the meeting then you have a connection.

- (f) I am a student because I attend university. **Solution:**

$p$ : I am a student

$q$ : I attend university

$$q \rightarrow p$$

This can be understood that *since* I attend university, then necessarily I am a student. You could also be a student if you do not attend a university, for example if you attend high school, but necessarily if you attend university then you must be a student.

### 3. I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

- (a)  $p$ : The sun is out  
 $q$ : We have class outside

$$p \rightarrow q$$

**Solution:**

If the sun is out, then we have class outside.  
Whenever the sun is out, we have class outside.

- (b)  $p$ : the book has been out for a week.  
 $q$ : I don't have homework.  
 $r$ : I have finished reading the book.

$$(p \wedge q) \rightarrow r$$

**Solution:**

I have finished reading the book, if it has been out for a week and I don't have homework.  
The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

- (c)  $p$ : I have read the manual  
 $q$ : I operate the machine

$$q \rightarrow p$$

**Solution:**

I operate the machine only if I have read the manual.  
Operating the machine implies that I have read the manual.

### 4. Look Both Ways

For each of the following, define propositional variables and translate the sentences into logical notation.

- (a) Watching Naruto is necessary and sufficient to having a good weekend only if I got less than 7 hours of sleep.

**Solution:**

$p$  : I watched Naruto  
 $q$  : I had a good weekend  
 $r$  : I got less than 7 hours of sleep

$$(p \leftrightarrow q) \rightarrow r$$

- (b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

**Solution:**

$p$  : Berries are ripe along the trail  
 $q$  : Hiking is safe  
 $r$  : Grizzly bears have not been seen in the area

$$p \rightarrow (q \leftrightarrow r)$$

(c) Unless I am trying to type something, my cat is either eating or sleeping. **Solution:**

$p$  : I am trying to type something  
 $q$  : My cat is eating  
 $r$  : My cat is sleeping

$$\neg p \rightarrow (q \oplus r)$$

## 5. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation. **Solution:**

$p$  : I am drinking tea  
 $q$  : I am eating a cookie

$$(p \rightarrow q) \vee (q \rightarrow p)$$

(b) Fill out a truth table for your expression. **Solution:**

$p$	$q$	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

## 6. Exclusive Or

Exclusive or ( $\oplus$ ) and inclusive or ( $\vee$ ) both can be translated as “or” in English. For each of the following ambiguous phrases, decide which type of “or” is likely meant and why, then translate the sentence to symbolic logic.

(a) Experience with C or Java is required. **Solution:**

Inclusive or. Experience with both is usually not a bad thing.

$p$ : Experience with C is required

$q$ : Experience with Java is required

$$p \vee q$$

(b) Lunch includes soup or salad. **Solution:**

Exclusive or. Most restaurants charge you more for both.

$p$ : Lunch includes soup

$q$ : Lunch includes salad

$$p \oplus q$$

(c) Publish or perish. **Solution:**

This phrase is a common one among researchers – it means publish papers or your career will perish.

Exclusive or is meant; i.e. if you do indeed publish you should avoid the loss of your career.

$p$ : One must publish

$q$ : One will perish

$$p \oplus q$$

(d) To enter the country, you need a passport or voter registration card. **Solution:**

Inclusive or – if you have both, they won't kick you out!

$p$ : To enter the country, you need a passport

$q$ : To enter the country, you need a voter registration card

$$p \vee q$$