

# Translation Tips

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We're going to need to translate English sentences into formal logic (and back) to make them easier to work with. With practice, one can sometimes do the translation "automatically." Sometimes, though, it can help to have a process to fall back on. Let's practice it slowly.

## 1. Process outline

We will use this process:

1. Identify connecting words, convert all others to propositional variables.
2. Decide which connectors have the lowest priority and add parentheses accordingly.
3. Continue translating until all connectors are symbolic.
4. Verify the translation fits your intuition.

Let's try it on a complicated sentence:

**"We will take the light rail or the bus, or if you have your phone then we can take a Lyft and I'll Venmo you."**

1. Identify the connecting words, convert all others to propositional variables.

Let's start by highlighting the connecting words:

**"We will take the light rail **or** the bus, **or if** you have your phone **then** we can take a Lyft **and** I'll Venmo you."**

Then, we can define our propositional variables:

- $p$ : We will take the light rail.
  - $q$ : We will take the bus.
  - $r$ : You have your phone.
  - $s$ : We can take a Lyft.
  - $t$ : I will Venmo you.
2. Decide which connectors have the lowest priority and add parentheses accordingly.

By low priority, we mean the connectors you apply last. So, in  $(a + b) - c$ , "+" has high priority, "-" has low priority.

This step is the trickiest — where do we add the parentheses? Some rules of thumb:

- (i) Omitted words usually mean high priority.
- (ii) Commas appear near low-priority operators.
- (iii) Implications often have low priority.
- (iv) When in doubt, think about which parsing of the sentence makes more sense.

Let's try it! Recall our sentence looks like this:

**"We will take the light rail **or** the bus, **or if** you have your phone **then** we can take a Lyft **and** I'll Venmo you."**

"The bus" has omitted words ("we will take" is omitted)— that first "or" is probably very high priority.

The second "or" connects two complete ideas (that is we aren't omitting words near that or), and there's a comma right by it. That seems like a good candidate for the highest priority connector. Let's plug that in.

**“(We will take the light rail or the bus, )  $\vee$  (if you have your phone then we can take a Lyft and I’ll Venmo you.)”**

3. Continue translating until all connectors are symbolic.

The left parentheses now has only one connector, so let’s plug in the symbolic version there. Notice that this “or” has very high priority (connecting two simple propositions), as we predicted.

**“( $p \vee q$ ) $\vee$  (if you have your phone then we can take a Lyft and I’ll Venmo you.)”**

What do we do with the last term? There are two possible parsings:  $r \rightarrow (s \wedge t)$  and  $(r \rightarrow s) \wedge t$ . Which should have higher priority? We’ll go with the first parsing, for two reasons:

- (i) Implications usually have lower priority.
- (ii) Based on our last guideline, it makes more sense that Venmo would be connected to a Lyft ride,

Our final parsing:

$(p \vee q) \vee [r \rightarrow (s \wedge t)]$

4. Verify the translation meets our intuition. Let’s see what this says:

**“We can take one of two public transit options, or if we have the right tool, we can use some phone apps instead.”**

That’s a pretty reasonable sentence. The implication has a cause-effect flavor (which isn’t required, but a good sign) and the sentence has an internal logic.

We’ll call this our translation.

One warning: these rules of thumb are called that for a reason — they are just guidelines, not perfect rules that always work. English sentences often have more than one possible parsing. On homeworks and tests, we will do our best to make sure one interpretation is clearly better than the others. However, people aren’t perfect at communication, so it’s good to practice parsing through ambiguity.

## 2. Translating the other way

What if we genuinely **wanted** another meaning? What if we had the sentence  $[(p \vee q) \vee (r \rightarrow s)] \wedge t$ . How do we communicate that in English?

If we just translate piece-by-piece, we’ll probably get something like our original English sentence... which we just said parsed another way. How can we **force** the final “and” to have lower priority?

The answer is punctuation — add a period and make it a separate sentence. And/or reorder the propositions to get the desired effect. For example:

**“I’ll Venmo you. We will take the light rail or the bus, or if you have your phone then we can take a Lyft.”**

Or, use words to make it even clearer:

**“No matter what, I’ll Venmo you. We will take the light rail or the bus, or if you have your phone then we can take a Lyft.”**

Of course, the English sentences aren’t perfect, you could still come up with other parsings — that’s why we have the symbolic language!

## 3. Implication Varieties

Implications are extremely common. So we have **a lot** of ways to refer to them.

How do we translate with all of these varieties? The hardest part is deciding which part is  $p$  and which is  $q$ .

**Translations of  $p \rightarrow q$ .**

- $p$  implies  $q$ .
- whenever  $p$  is true  $q$  must be true.
- if  $p$  then  $q$ .
- $q$  if  $p$ .
- $p$  is sufficient for  $q$ .
- $p$  only if  $q$ .
- $q$  is necessary for  $p$ .

The list above can help you remember which part is the “hypothesis” and which part is the “conclusion.”

Another good strategy is to plug in “it is raining” and “I have my umbrella” (or your other favorite real-life implication) and see which sentence makes sense:

- It is raining only if I have my umbrella. **True**, I can’t get wet.
- I have my umbrella only if it is raining. **False**, I take it whenever there’s at least a 30% chance of rain, sometimes it doesn’t rain.

Biconditionals also have a lot of possible translations. Luckily since  $p \leftrightarrow q$  and  $q \leftrightarrow p$  are equivalent propositions, we don’t need to worry about which is which.