CSE 311: Foundations of Computing I

Induction Practice

Review

5 Steps to an Induction Proof: .

1. "Let $P(n)$ be $\langle$fill in$\rangle$. We will show that $P(n)$ is true for every $n \in \mathbb{N}$ (or equivalently integer $n \geqslant 0$) by induction."
2. "Base Case:" Prove $P(0)$
3. "Inductive Hypothesis: Suppose $P(k)$ is true for some arbitrary integer $k \geqslant 0$"
4. "Inductive Step:" Prove that $P(k+1)$ is true.

   Use the goal to figure out what you need.
   Make sure you are using I.H. and point out where you are using it.
   (Don't assume $P(k+1)$!)
5. "Conclusion: The claim holds by the principle of induction for"

Task 1 – Sums!

Let a be an integer with $a > 1$. Prove, by induction, that

$$\sum_{i=0}^{n} a^i < \frac{a^{n+1}}{a-1}$$

for all integers $n \geqslant 0$.

Task 2 – Fibonacci Numbers

Define the Fibonacci numbers as follows:

$$f(0) = 1$$

$$f(1) = 1$$

$$f(n) = f(n-1) + f(n-2) \quad \text{for all } n \in \mathbb{N},\ n \geqslant 2.$$

Show that $f(n) \geqslant 2^{n/2}$ for all $n \geqslant 2$ by induction.

Task 3 – Sharing Chocolate

Given a rectangular chocolate bar comprised of n individual squares, find and prove the number of breaks required to split the bar into its individual squares.

Note: There are no restrictions on the dimensions of the chocolate bar, only that it has n total squares of chocolate. Additionally, each break can only split one piece of chocolate into two separate pieces.