Countable

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The set A is countable iff there is an injection from A to $\mathbb{N}$, Equivalently, A is countable iff it is finite or there is a bijection from A to $\mathbb{N}$

$\mathbb{N}, \mathbb{Z}, \{x: x$ is an even integer$\}$ are all countable.

To build a bijection from A to $\mathbb{N}$, just list all the elements!

Bijection

One-to-one (aka injection)

A function f is one-to-one iff $\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff $\forall b \in B \exists a \in A(b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff f is one-to-one and onto

A bijection maps every element of the domain to **exactly** one element of the co-domain, and every element of the domain to **exactly** one element of the domain.

Proof that $[0,1)$ is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	…
$f(0)$	0.	3	3	3	3	3	3	3	3	…
$f(1)$	0.	2	7	2	7	2	8	5	4	…
$f(2)$	0.	1	4	1	5	9	2	6	5	…
$f(3)$	0.	2	2	2	2	2	2	2	2	…
$f(4)$	0.	1	2	3	4	5	6	7	8	…
$f(5)$	0.	9	8	7	6	5	4	3	2	…
$f(6)$	0.	8	2	7	6	4	5	7	4	…
$f(7)$	0.	5	9	4	2	7	5	1	7	…
…	…	…	…	…	…	…	…	…	…	…

Let's Do Another!

Let $B = \{0,1\}$. Call a function $g: \mathbb{N} \to B$ a "binary valued function"

Intuitively, g would be something like
public boolean g(BigInteger input){ }

If we could write that g in Java.

How many possible $g: \mathbb{N} \to B$ are there?