

A Proof Outline

Claim: $\{0^k 1^k : k \geq 0\}$ is an irregular language.

...

Let $S = [\text{TODO}]$. *S is an infinite set of strings.*

Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state. *We don't get to choose x, y*

Consider the string $z = [\text{TODO}]$ *We do get to choose z depending on x, y*

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M . Observe that $xz \in \{0^k 1^k : k \geq 0\}$ but $yz \notin \{0^k 1^k : k \geq 0\}$. Since q can be only one of an accept or reject state, M does not actually recognize $\{0^k 1^k : k \geq 0\}$. That's a contradiction!

Therefore, $\{0^k 1^k : k \geq 0\}$ is an irregular language.

Irregularity Proof Template

1. Suppose for the sake of contradiction that L is regular. Then there is some DFA M that recognizes L .
2. Let S be _____
3. Because the DFA is finite and S is infinite, there are two (different) strings $x = \underline{\hspace{2cm}}$ and $y = \underline{\hspace{2cm}}$ in S such that x and y go to the same state when read by M .
4. Consider the string $z = \underline{\hspace{2cm}}$.
5. Since x, y both end up in the same state, and we appended the same z , both $xz = \underline{\hspace{2cm}}$ and $yz = \underline{\hspace{2cm}}$ end up in the same state of M . Since $xz \in L$ and $yz \notin L$, M does not recognize L . But that's a contradiction!
6. So L must be an irregular language.

Let's Try another

The set of strings with balanced parentheses is not regular.

What do you want S to be? What would you have to count?

The number of unclosed parentheses.

Let $S = \dots$

One more, just the key steps

What about $\{a^k b^k c^k : k \geq 0\}$?