Basis: $0 \in T,\ 5 \in T$
Recursive: If $x, y \in T$ then $x + y \in T$ and $x - y \in T$.

Claim: $5|x$ for all $x \in T$.

Let $P(x)$ be "$5|x$." We prove $P(x)$ holds for all $x \in T$ by structural induction.
Base Case ( ):
Inductive Hypothesis:
Inductive Step:

Thus $\mathrm{P}(\mathrm{x})$ holds for all $x \in T$ by structural induction

Claim for all $x, y \in \Sigma^*$ len(x·y)=len(x) + len(y).

Let $P(y)$ be "len(x·y)=len(x) + len(y) for all $x \in \Sigma^*$."

We prove $P(y)$ for all $\mathrm{y} \in \Sigma^*$ by structural induction.

Base Case:

Inductive Hypothesis

Inductive Step:

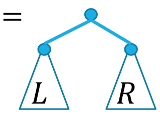
We conclude that $P(y)$ holds for all string y by the principle of induction. Unwrapping the definition of P, we get $\forall x \forall y \in \Sigma^*$ len(xy)=len(x)+len(y), as required.

Practice: Structural Induction on Trees

Let $P(T)$ be "$\text{leaves}(T) \geq \frac{\text{size}(T)}{2} + \frac{1}{2}$ ". We show $P(T)$ for all binary trees T by structural induction.

Base Case: Let $T =$  .

Inductive Hypothesis: Suppose $\mathrm{P}(L)$ and $\mathrm{P}(R)$ hold for arbitrary trees L, R. Let $T =$



So $P(T)$ holds, and we have $P(T)$ for all binary trees T by the principle of induction.

Induction: Hats!

Define $P(n)$ to be "in every line of n people with gold and purple hats, with a purple hat at one end and a gold hat at the other, there is a person with a purple hat next to someone with a gold hat"

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case: $n = 2$

Inductive Hypothesis:

Inductive Step:

By the principle of induction, we have $P(n)$ for all $n \geq 2$