Recursive Definitions of Sets

Q1: What is this set?

Basis Step: $6 \in S, 15 \in S$

Recursive Step: If $x, y \in S$ then $x + y \in S$

Q2: Write a recursive definition for the set of powers of 3 $\{1,3,9,27, \dots\}$

Basis Step:

Recursive Step:

Structural Induction Template

1. Define $P()$ State that you will show $P(x)$ holds for all $x \in S$ and that your proof is by structural induction.

2. Base Case: Show $P(b)$
[Do that for every b in the basis step of defining S]

3. Inductive Hypothesis: Suppose $P(x)$
[Do that for every x listed as already in S in the recursive rules].

4. Inductive Step: Show $P()$ holds for the "new elements."
[You will need a separate step for every element created by the recursive rules].

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

$\text{len}(\varepsilon) = 0$ $\text{len}(wa) = \text{len}(w) + 1$

$\varepsilon^R = \varepsilon$ $(wa)^R = aw^R$

Basis: $\varepsilon \in \Sigma^*$ Recursive: If $w \in \Sigma^*$ and $a \in \Sigma$, then $wa \in \Sigma^*$

Claim: For any string $s \in \Sigma^*$, $\text{len}(s^R) = \text{len}(s)$

Let $P(s)$ be "$\text{len}(s^R) = \text{len}(s)$." We prove $P(s)$ holds for all strings s by structural induction.

Base Case ($s = \varepsilon$): Since $\varepsilon^R = \varepsilon$, $\text{len}(\varepsilon^R) = \text{len}(\varepsilon) = 0$. RHS: $\text{len}(\varepsilon) = 0$. Since $0 = 0$, the base case holds.

Inductive Hypothesis: Suppose $P(w)$ for some arbitrary string w.

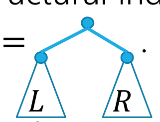
Inductive Step:

So $P(w)$ holds.

Thus $\text{P}(s)$ holds for all strings s by structural induction

Structural Induction on Binary Trees (cont.)

Let $P(T)$ be "size(T) $\leq 2^{height(T)+1} - 1$". We show $P(T)$ for all binary trees T by structural induction.

$T =$  .

height(T)$=1 + \max\{height(L), height(R)\}$

size(T)$=$ 1 +size(L)+size(R)

So $P(T)$ holds, and we have $P(T)$ for all binary trees T by the principle of induction.