How do we know recursion works?

```
//Assume i is a nonnegative integer
//returns 2^i.
public int CalculatesTwoToTheI(int i){
     if(i == 0)
          return 1;
     else
          return 2*CaclulatesTwoToTheI(i-1);
}
```

Why does `CalculatesTwoToTheI(4)` calculate `2^4`?
Convince the people around you!

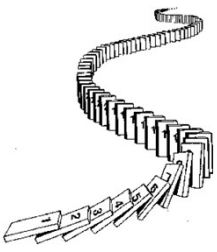
The Principle of Mathematical Induction

$$\mathrm{P}(0) \wedge \forall k \left(\mathrm{P}(k) \rightarrow \mathrm{P}(k+1)\right)$$

<u>Base Case</u>
Prove $\mathrm{P}(0)$ holds.

<u>Inductive Hypothesis</u>
Let $k \geq 0$ be an arbitrary integer.
Suppose $\mathrm{P}(k)$ holds.

<u>Inductive Step</u>
Prove that $\mathrm{P}(k+1)$ holds (using $\mathrm{P}(k)$)



Making Induction Proofs Pretty

All of our induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.
2. Show $P(0)$ i.e. show the base case
3. Suppose $P(k)$ for an arbitrary k.
4. Show $P(k+1)$ (i.e. get $P(k) \rightarrow P(k+1)$)
5. Conclude by saying $P(n)$ is true for all n by induction.

Show that $\sum_{i=0}^{n} 2^i = 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$.

Let $P(n)$ be "$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$." We prove $P(n)$ holds holds for all integers $n \geq 0$ by induction on n.

Base Case ($k = 0$)

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq 0$.

Inductive Step:

So $P(k+1)$ holds.

Therefore $P(n)$ holds for all $n \geq 0$ by the principle of induction.