Two Claims

Determine if the following claims are true or false.

Claim 1: For all sets A, B, C, if $A \subseteq (B \cup C)$ then $A \subseteq B$ or $A \subseteq C$.

Claim 2: For all sets A, B, C it holds that $A \cap B \cap C \subseteq A \cup B$.

Exercises

Which of the following statements are true?

If $x \in A \cap B$ then $(x \in A) \cap (x \in B)$.

If $x \in C \backslash D$ then $x \in C \wedge \neg(x \in D)$.

If $X \subseteq \mathcal{P}(A)$ then $X \in A$.

If $(a, b) \in E \times F$ then $a \in E$ and $b \in F$.

Claim 3 (DeMorgan's Law for Sets)

Claim 3: For all sets A, B, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

Definitions
$A = B \equiv A \subseteq B \wedge B \subseteq A$
$A \cup B = \{x : x \in A \vee x \in B\}$
$A \cap B = \{x : x \in A \wedge x \in B\}$
$\bar{A} = \{x \in \mathcal{U} : x \notin A\}$

Proof by Contradiction

Proof by contradiction is a strategy for proving statements of any form.

The strategy to prove p is to assume $\neg p$ and derive $\mathbf{False}$. (i.e. ($\neg$claim) $\rightarrow$ F)

- E.g. the strategy to prove $p \rightarrow q$ is to assume $p \wedge \neg q$ and derive $\mathbf{False}$.
- E.g. the strategy to prove $p \vee q$ is to assume $\neg p \wedge \neg q$ and derive $\mathbf{False}$.