## Integer

We need a basic starting point to be able to prove things.

Objects to work with.

**An integer: is any real number with no fractional part.**

Some **definitions** to analyze

**Even**

`Even(x)` := An integer, $x$, is even if and only if there is an integer $k$ such that $x = 2k$.

**Odd**

`Odd(x)` := An integer, $x$, is odd if and only if there is an integer $k$ such that $x = 2k + 1$.

## Direct Proof Steps

These are the usual steps. We'll see different outlines in the future!!

- Introduction
  - Declare an arbitrary variable for each ∀ quantifier
  - Assume the left side of the implication
- Core of the proof
  - Unroll the predicate definitions
  - Manipulate towards the goal (using creativity, algebra, etc.)
  - Reroll definitions into the right side of the implication
- Conclude that you have proved the claim

## Another Direct Proof

Definitions
$\mathrm{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: "The product of two odd integers is odd."

$$\forall x \forall y \left( \left( \mathrm{Odd}(x) \land \mathrm{Odd}(y) \right) \rightarrow \mathrm{Odd}(xy) \right)$$

## Proof by Contrapositive

Definitions
$\mathrm{Odd}(x) \coloneqq \exists k(x = 2k + 1)$

Prove: For an integer $x$, if $3x + 2$ is odd, then $x$ is odd.