

Warm Up

Translate this sentence into symbolic logic, and describe a weather pattern and transportation method that causes the proposition to be false.

It is snowing today, and if it is raining or snowing then we won't walk to school.

Properties of Logical Connectives

These identities hold for all propositions p, q, r

- **Identity**
 - $p \wedge T \equiv p$
 - $p \vee F \equiv p$
- **Domination**
 - $p \vee T \equiv T$
 - $p \wedge F \equiv F$
- **Idempotent**
 - $p \vee p \equiv p$
 - $p \wedge p \equiv p$
- **Commutative**
 - $p \vee q \equiv q \vee p$
 - $p \wedge q \equiv q \wedge p$
- **Associative**
 - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
 - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
 - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
 - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
 - $p \vee (p \wedge q) \equiv p$
 - $p \wedge (p \vee q) \equiv p$
- **Negation**
 - $p \vee \neg p \equiv T$
 - $p \wedge \neg p \equiv F$
- **DeMorgan's Laws**
 - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
 - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- **Double Negation**
 - $\neg\neg p \equiv p$
- **Law of Implication**
 - $p \rightarrow q \equiv \neg p \vee q$
- **Contrapositive**
 - $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Our First Proof

$$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$$

None of the rules look like this

Practice of Proof-Writing:

Big Picture... WHY do we think this might be true?

The last two "pieces" came from the $\equiv (\neg a \vee b)$ vacuous proof lines...maybe the " $\neg a$ " came from there? Maybe that **simplifies** down to $\neg a$

Converse, Contrapositive

Implication:

If it's raining, then I have my umbrella.

$$p \rightarrow q$$

Contrapositive:

$\neg q \rightarrow \neg p$ If I don't have my umbrella, then it is not raining.

Converse:

If I have my umbrella, then it is raining.

$$q \rightarrow p$$

Inverse:

$\neg p \rightarrow \neg q$ If it is not raining, then I don't have my umbrella.

How do these relate to each other?

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T						
T	F						
F	T						
F	F						