CSE 311 : Summer 2025 Midterm Exam Solutions

Name:	**NetID:** @uw.edu

Instructions

- You have sixty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.
- For multiple choice questions, options are shown in ◯ circles; completely fill in the circle for the (one) best answer.

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- All proofs for this exam must be English proofs.
- If you don't initially know how to approach a proof, it's often helpful to write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

1. Who Will Save You Now?

Interpret all sentences below as being in "mathematical English." You may use the following predicates; the definition for the predicate is given after the colon in the list below.

- $\texttt{Superhero}(x)$: x is a superhero
- $\texttt{Villain}(x)$: x is a villain
- $\texttt{Civilian}(x)$: x is a civilian
- $\texttt{WouldSave}(x, y)$: x would save y
- $\texttt{IsBald}(x)$: x is bald
- $\texttt{IsPlotting}(x, y)$: x is plotting against y

For the following questions, let the domain of discourse be people. Use $x = y$ to state that two people are the same, and $x \neq y$ to state that two people are different.

Translate the following English sentences in parts (a) and (b) into predicate logic.

(a) There is some bald villain that no superhero would save.

Solution:

$\exists x \forall y\, (\text{Villain}(x) \land \text{IsBald}(x) \land \text{Superhero}(y) \rightarrow \neg\text{WouldSave}(y, x))$

(b) A superhero is bald only if a villain is also bald and plotting against some civilian.

Solution:

$\forall x \exists y \exists z\, ((\text{Superhero}(x) \land \text{IsBald}(x)) \rightarrow (\text{Villain}(y) \land \text{Civilian}(z) \land \text{IsBald}(y) \land \text{isPlotting}(y, z)))$

Translate the predicate logic statements in parts (c) and (d) into English. Your English translation must take advantage of domain restriction where possible.

(c) $\forall x \forall y ((\texttt{Superhero}(x) \rightarrow \neg\, \texttt{WouldSave}(x, x)) \land (\texttt{Superhero}(x) \land \texttt{Villain}(y) \land \neg\, \texttt{IsBald}(y) \rightarrow \texttt{WouldSave}(x, y)))$

Solution:

A superhero wouldn't save themself, but they would save a villain if they weren't bald.

(d) $\exists x \exists y ((x \neq y \land \texttt{IsPlotting}(x, y) \land \texttt{IsPlotting}(y, x)))$

Solution:

There are at least two different people plotting against each other.

2. (Mod)ern Relationships

Prove the following claim:

For all integers $n > 0$ and $x, y \in \mathbb{Z}$: if $x \equiv 2 \pmod{4n}$ and $y \equiv 2 \pmod{3n}$, then $3x + 4y \equiv 14 \pmod{12n}$.

Hint: Use the definitions. **Solution:**

Let n be an arbitrary integer. Suppose $n > 0$. Let x and y be arbitrary integers. Suppose $x \equiv 2 \pmod{4n}$ and $y \equiv 2 \pmod{3n}$. By the definition of congruence we have $4n|(x-2)$ and $3n|(y-2)$. By the definition of divides there exists integers k and m such that $x - 2 = 4nk$ and $y - 2 = 3nm$. If we multiply the first equation by 3 we have $3x - 6 = 12nk$. If we multiply the second equation by 4 we have $4y - 8 = 12nm$. Adding them we get $3x + 4y - 14 = 12nk + 12nm$. Thus by the definition of divides $12n|(3x+4y) - 14$. By the definition of congruence we have $3x + 4y \equiv 14 \pmod{12n}$. Since n, x, y were arbitrary, our claim holds for all integers $n > 0$.

3. Brain Power

Prove the following claim:

For all sets S and T, $\mathcal{P}(S) \cup \mathcal{P}(T) \subseteq \mathcal{P}(S \cup T)$.

Your proof must be in **English**. Do not write a logical equivalences proof. You can still use symbols within your **English proof** where appropriate.

Hint: At some point in the proof you may want to use cases.

Solution:

Let S, T be arbitrary sets. Let X be an arbitrary element of $\mathcal{P}(S) \cup \mathcal{P}(T)$. By the definition of union, $X \in \mathcal{P}(S)$ or $X \in \mathcal{P}(T)$. Then there are 2 cases:

Case 1: $X \in \mathcal{P}(S)$. Then, by the definition of power set, $X \subseteq S$.
Let x be an arbitrary element of X. By definition of subset, $x \in S$.
Then certainly $x \in S$ or $x \in T$.
So, by definition of union, $x \in (S \cup T)$.
Since x was an arbitrary element of X, by the definition of subset, we have $X \subseteq (S \cup T)$.
Then, by definition of power set, $X \in \mathcal{P}(S \cup T)$.

Case 2: $X \in \mathcal{P}(T)$. Then, by the definition of power set, $X \subseteq T$.
Let x be an arbitrary element of X. By definition of subset, $x \in T$.
Then certainly $x \in S$ or $x \in T$.
By definition of union, $x \in (S \cup T)$.
Since x was an arbitrary element of X, by the definition of subset, we have $X \subseteq (S \cup T)$.
Then, by definition of power set, $X \in \mathcal{P}(S \cup T)$.

Since these cases are exhaustive, $X \in \mathcal{P}(S \cup T)$. Since X is arbitrary, by definition of subset, we've shown that $\mathcal{P}(S) \cup \mathcal{P}(T) \subseteq \mathcal{P}(S \cup T)$. Since S, T were arbitrary sets, the claim holds for all sets.

4. Horrible Big O-Bounds

Prove the following claim by induction:

For all $n \in \mathbb{N}$, $n! \leq n^n$.

where $n! = 1 \cdot 2 \ldots (n-1) \cdot n = \prod_{i=1}^{n} i$. We define $0^0 = 1$ and $0! = 1$.
Note: Make sure to use the template covered in class, including defining a predicate $P()$. **Solution:**

Let $P(n)$ be the claim: "$n! \leq n^n$". We will prove $P(n)$ by induction on all integers $n \geq 0$.

Base Case ($n = 0$): Since $0! = 1$ and $0^0 = 1$, $0! \leq 0^0$ and therefore $P(0)$ holds.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary integer $k \geq 0$.

Inductive Step: Goal: $P(k+1)$

$$\begin{aligned}
(k+1)! &= (k+1)k! \\
&\leq (k+1)k^k && \text{[from IH]} \\
&\leq (k+1)(k+1)^k \\
&= (k+1)^{k+1}
\end{aligned}$$

Thus $P(k+1)$ holds. Since k was arbitrary the claim holds by the principle of induction for all integers $n \in \mathbb{N}$.

5. Short Answer

(a) Which of the following is a (modular) multiplicative inverse of $2 \pmod 5$?

◯ 1
◯ 2
◯ 3
◯ 4

Solution:

Third option. We are looking for an integer x such that $2x \equiv 1 \mod 5$. We can check each option to get that $2(3) = 6$, where $6 \equiv 1 \pmod 5$, so $x = 3$.

(b) Consider the boolean algebra statement $(p+q)' \cdot q'$. Which of the following is an equivalent statement?

◯ $p' + q'$
◯ $p' \cdot q$
◯ $(p' + q') \cdot q'$
◯ $(p+q)'$

Solution:

Fourth option

(c) You are trying to prove the following claim:

For all integers x, if x^3 is odd, then x is odd.

For the following questions, you should introduce all variables and necessary assumptions, but do not apply any definitions or theorems.

(i) Suppose you want to prove by contrapositive. Write the first 1-2 sentences of your proof by contrapositive.

(ii) Suppose you want to prove by contradiction. Write the first 1-2 sentences of your proof by contradiction.

Solution:

(i) Let x be an arbitrary integer. Suppose x is even.

(ii) Let x be an arbitrary integer. Suppose for the sake of contradiction that x^3 is odd and x is even.
OR
Suppose for the sake of contradiction that there exists some integer x where x^3 is odd and x is even.

(d) Your friend, who is on a quest to understand what a blue raspberry is, tells you the following statement is true: "If I eat blueberries, or I don't eat kiwis, then I will turn into a blue raspberry." What can you conclude about the statement "I will not turn into a blue raspberry only if I don't eat blueberries and I eat kiwis"?

◯ The second statement must be true.
◯ The second statement cannot be true.
◯ The second statement might or might not be true.

Solution:

Answer choice 1

(e) Which of the following is the **DNF** of the following truth table?

◯ $(p \land q \land r) \lor (p \land q \land \neg r) \lor (p \land \neg q \land r)$
◯ $(p \land \neg q \land r) \lor (\neg p \land q \land r) \lor (\neg p \land \neg q \land \neg r)$
◯ $(p \land \neg q \land r) \land (\neg p \land q \land r) \land (\neg p \land \neg q \land \neg r)$
◯ $(p \lor \neg q \lor r) \land (\neg p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$

p	q	r	$G(p, q, r)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

Solution:

2nd option