CSE 311 : Summer 2025 Midterm Retake Exam Solutions

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Instructions

- You have sixty minutes to complete this exam.
- You are permitted one piece of 8.5x11 inch paper with handwritten notes (notes are allowed on both sides of the paper).
- You may not use a calculator or any other electronic devices during the exam.
- We will be scanning your exams before grading them. Please write legibly, and avoid writing up to the edge of the paper.
- **Problems are printed on both the front and back of each page!**
- You may also use the last page for extra space, but tell us where to find your answer if it's not right below the problem.
- If you want us to grade something you wrote on scratch paper, put your name and netid on the paper and tell us when you turn in your exam that you have an extra sheet.
- For multiple choice questions:
  - If options are shown in ◯ circles, completely fill in the circle for the (one) best answer.
  - If options are shown in ☐ squares, completely fill in the squares for **ALL** correct answers (there may be more than one).

Advice

- Remember to properly format English proofs (e.g. introduce all your variables).
- All proofs for this exam must be English proofs.
- If you don't initially know how to approach a proof, it's often helpful to write the start of the proof and put the "target" and conclusion at the bottom.
- Remember to take deep breaths.

1. When Pigs Fly...

Interpret all sentences below as being in "mathematical English." You may use the following predicates; the definition for the predicate is given after the colon in the list below.

- `Bird`(x) : x is a bird
- `Pig`(x) : x is a pig
- `WillDestroy`(x, y) : x will destroy y
- `IsAngry`(x) : x is angry
- `CanFly`(x) : x can fly

For the following questions, let the domain of discourse be animals. Use $x = y$ to state that two animals are the same, and $x \neq y$ to state that two animals are different.

Translate the following English sentences in parts (a) and (b) into predicate logic.

(a) Pigs can fly unless birds can fly.

Solution:

$\forall x \forall y((\text{Bird}(x) \land \text{Pig}(y) \land \neg\text{CanFly}(x)) \to \text{CanFly}(y))$

(b) Exactly one angry pig can fly.

Solution:

$\exists x\,((\text{Pig}(x) \land \text{IsAngry}(x) \land \text{CanFly}(x)) \land \forall y(\text{Pig}(y) \land \text{IsAngry}(y) \land \text{CanFly}(y) \to y = x))$

Translate the predicate logic statements in parts (c) and (d) into English. Your English translation must take advantage of domain restriction where possible.

(c) $\forall x \forall y\,([\text{Pig}(x) \land \text{Bird}(y)] \to (\text{WillDestroy}(x, y) \leftrightarrow [\neg\text{CanFly}(y) \land \neg\text{IsAngry}(y)]))$

Solution:

Pigs will destroy birds if and only if the birds can't fly and aren't angry.

Birds can't fly and aren't angry if and only if pigs will destroy them.

(d) $\forall x \forall y\,([\text{Pig}(y) \land \text{Bird}(x) \land \text{WillDestroy}(x, y)] \to [\text{IsAngry}(x) \to \text{CanFly}(y)])$

Solution:

If a bird will destroy a pig, the bird is angry only if the pig can fly.

2. (Mod)ern Relationships

Prove the following claim:

For all integer a, $a^2 \equiv 0 \pmod 4$ or $a^2 \equiv 1 \pmod 4$.

Hint: Try some examples and use cases! **Solution:**

Let a be an arbitrary integer. Then a is either even or odd. Let us do this by cases.
Case 1: a **is even**
By the definition of even, there exists an integer k such that $a = 2k$. Then $a^2 = 4k^2$. Since k was an integer and integers are closed under multiplication $a^2 \equiv 0 \pmod 4$ by the definition of congruence.
Case 2: a **is odd**
By the definition of odd, there exists an integer j such that $a = 2j + 1$. Then $a^2 = (2j + 1)^2 = 4j^2 + 4j + 1 = 4(j^2 + j) + 1$. Since j was an integer and integers are closed under multiplication we have $a^2 \equiv 1 \pmod 4$.
Since our cases were exhaustive and exclusive we have shown that our claim holds.

3. cArts and Crafts

Prove the following claim:

For all sets A, B, C, D, if $A \subseteq C$ and $B \subseteq D$ then $(A \times B) \subseteq (C \times D)$

Your proof must be in **English**. Do not write a logical equivalences proof. You can still use symbols within your **English proof** where appropriate.

Solution:

Let A, B, C, D be arbitrary sets. Suppose $A \subseteq C$ and $B \subseteq D$.
Let (y, z) be an arbitrary element of $(A \times B)$.
By definition of Cartesian Product, $y \in A$ and $z \in B$.
By definition of subset, $y \in C$ and $z \in D$.
By definition of Cartesian Product, $(y, z) \in (C \times D)$
Since (y, z) was an arbitrary element of $(A \times B)$, by the definition of subset, we have $(A \times B) \subseteq (C \times D)$
Since A, B, C, D were arbitrary sets, the claim holds for all sets.

4. Star Power!

We define the following function Mystery(n) as follows:

```
Mystery(n) {
  if (n == 1) return 1
  else {
    return Mystery(n-1) + 12(n-1)
  }
}
```

Prove that, for all integers $n \geq 1$,

$$\texttt{Mystery(n)} = 6n(n-1) + 1.$$

Note: Make sure to use the template covered in class, including defining a predicate $P()$. **Solution:**

Let $P(n)$ be the claim: "$\texttt{Mystery(n)} = 6n(n-1)+1$". We will prove $P(n)$ by induction on all integers $n \geq 1$.

Base Case ($n = 1$): Since $\texttt{Mystery(1)} = 1$ and $6(1)(1-1)+1 = 0+1 = 1$, $1 = 1$ and therefore $P(1)$ holds.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary integer $k \geq 1$.

Inductive Step: Goal: $P(k+1)$

From the function, we know that $k+1 \geq 2$ so we will enter the else statement. In that case, we know that $\texttt{Mystery(k+1)} = \texttt{Mystery(k)} + 12(k)$. Then, we apply Inductive Hypothesis to get that:

$$\begin{aligned}
\texttt{Mystery(k)} + 12(k) &= 6k(k-1) + 1 + 12(k) && \text{[from IH]} \\
&= 6k^2 - 6k + 12k + 1 \\
&= 6k^2 + 6k + 1 \\
&= 6(k^2 + k) + 1 \\
&= 6(k+1)k + 1
\end{aligned}$$

Thus $P(k+1)$ holds. Since k was arbitrary the claim holds by the principle of induction for all integers $n \in \mathbb{N}$.

5. Short Answer

(a) Define the following propositions:

$p :=$ Woodstock is cold
$q :=$ It is snowing
$r :=$ Snoopy makes hot chocolate

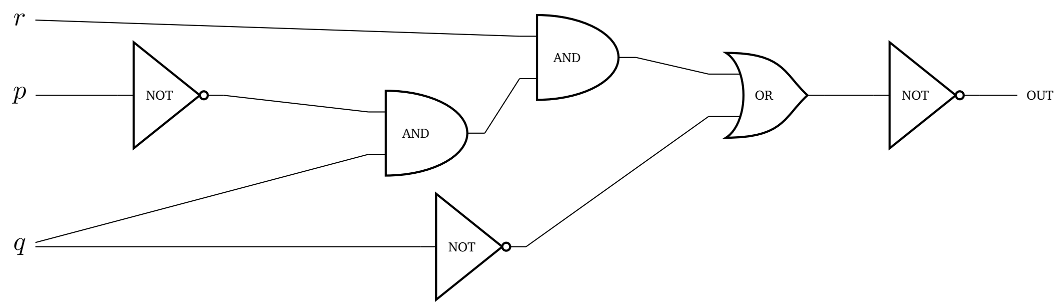
and consider the symbolic logic statement $p \wedge q \rightarrow r$. Which of the following is an equivalent English translation? Select ALL that apply.

☐ Snoopy makes hot chocolate if Woodstock is cold and it is snowing.
☐ Snoopy makes hot chocolate only if Woodstock is cold and it is snowing.
☐ Woodstock is cold, and if it is snowing then Snoopy will make hot chocolate.
☐ Snoopy makes hot chocolate, or Woodstock is not cold, or it isn't snowing.

Solution:

First and fourth option

(b) Which of the following is the circuit below equivalent to?



◯ $q \wedge (\neg r \vee (p \wedge \neg q))$
◯ $\neg(\neg q \vee ((\neg p \wedge q) \wedge r))$
◯ $\neg(\neg q \wedge ((p \vee q) \vee r))$
◯ $\neg(\neg q \vee ((\neg p \wedge \neg q) \wedge r))$ **Solution:**

Answer choice 2

(c) You want to prove the following claim:

For all integers x, if $x \equiv 1 \pmod{4}$, then $2 \nmid x$.

We will write the opening sentence for a proof by contradiction. **Fill in the blank:** "Suppose, for the sake of contradiction, that there is some integer x such that ________________."

◯ If $x \not\equiv 1 \pmod{4}$, then $2 \nmid x$.
◯ If $x \equiv 1 \pmod{4}$, then $2 \mid x$.
◯ $x \not\equiv 1 \pmod{4}$ and $2 \nmid x$.
◯ $x \equiv 1 \pmod{4}$ and $2 \mid x$.

Solution:

4th option

(d) Let our domain of discourse be integers. Consider the following claim: "$7 \mid x$ only if x is odd." This claim is:

◯ A tautology
◯ A contradiction
◯ Neither

Solution:

3rd option. Consider $x = 7$ (for which it is true) and $y = 14$ (false).

(e) Which of the following can the Extended Euclidean Algorithm be used for?

◯ Finding an integer b such that $ab \equiv 0 \pmod{m}$ for some integers a, m.
◯ Determining whether two integers a and b are prime.
◯ Finding the largest integer that divides two integers a and b.
◯ Finding the prime factors of an integer.

Solution:

3rd