

# Homework 1: Propositional Logic

---

Due date: Wednesday July 2 at 11:59 PM

If you work with others (and you should!), remember to follow the collaboration policy outlined in the [syllabus](#). In general, you are graded on both the clarity and accuracy of your work. Your solution should be clear enough that someone in the class who had not seen the problem before would understand it.

We sometimes describe approximately how long our explanations are. These are intended to help you understand approximately how much detail we are expecting. You are allowed to have longer explanations, but explanations significantly longer than necessary may receive deductions.

You should read the [grading guidelines](#) on the assignments page, but much of it will only be relevant with later homeworks.

## 1. Syllabus

Read the syllabus and/or watch “Lecture 1” on [panopto](#) until the 15:30 mark. If you have any questions about the syllabus, ask us on Ed!

On your homework submission, write “I have watched lecture 1 and/or read the syllabus, and agree to follow the collaboration and AI policies.” for full credit.

## 2. Heroes in Translation!

Translate the English statements into symbolic logic. You will need to define the propositions you use. Make sure the propositions you introduce are atomic (not the combination of smaller propositions).

*Make sure you translate each sentence as an implication.* Remember: You need subject and verb in each atomic proposition. Define them explicitly.

*Hint: remember propositions need to unambiguously assert something. In all propositions, you need a subject and verb—you may need to infer those from context, but be sure to include it explicitly in your propositions.*

- (a) The Batcave’s gates are open on weekdays and on days with a Justice League emergency.

*Hint: It’s possible a Justice League emergency happens on a weekday; in such cases, the gates are open.*

- (b) The Millennium Falcon doesn’t lift off only if the hyperdrive fails and there’s no coaxium fuel.

- (c) Define a set of *at most five* atomic propositions. Then, use them to translate all of these sentences into logical notation. *Do not simplify the statements.* Note that we want you to translate the sentences as they appear; you should not add any knowledge about the context in doing the translation.

Tony Stark is testing a new suit...

- (i) If the suit malfunctions on Monday and Tuesday, then Tony needs a new one.
- (ii) If Tony needs a new suit, but the suit didn’t malfunction on Monday, then he must have crashed it during training on Wednesday.
- (iii) Tony crashes the suit on Wednesday, if he needs a new one.

### 3. The Trials of Propositional Bending

The following sentences are idiomatic in English...but not very clear logically. Convert the statement into propositional logic (you'll need to define atomic propositions first), then write an English sentence that has the same meaning, but is clearer logically.

- (a) Unless Ba Sing Se is not under attack, if Sokka enters the Spirit Library and solves the riddle, Sokka can access the hidden scrolls.
- (b) Aang can return to the Air Temple, if the sky bisons are flying, and if his glider is fully repaired as well.

### 4. Inequivalence

For each part, find a truth assignment (i.e. an assignment of True or False to the variables) to show the pair of statements are not logically equivalent. Explain why your assignments work (our explanations are 1-2 sentences).

- (a)  $p \rightarrow (q \rightarrow r)$  vs.  $(p \rightarrow q) \rightarrow r$
- (b)  $\neg(p \vee q \vee r)$  vs.  $\neg p \vee \neg q \wedge \neg r$
- (c)  $a \vee (b \wedge c)$  vs.  $(a \vee b) \wedge c$

**Hint:** Use a truth table!

### 5. Compound Proposition

Find a compound proposition involving the variables  $p, q, r,$  and  $s$  that is true precisely when either

- at least three of  $p, q, r$  and  $s$  are true,
- or  $p$  and  $s$  are true and  $r$  is false

The “either...or...” above is an exclusive or; we would want the statement to evaluate to false if  $p, q, s$  are true and  $r$  is false since we want either at least 3 of the variables to be true or  $p, s$  to be true and  $r$  to be false.

In addition to writing an expression, explain why your answer works (1-2 sentences).

*For this problem, you may only use the logical connectives:  $\neg, \vee, \wedge, \rightarrow$*

**Hint:** There are a few different ways to approach this question, but the easiest is probably to think of all the settings which will meet the conditions and make a statement that properly combines the elements of your list.

### 6. Highly Illogical

After searching for months, you have finally tracked down Sherlock Holmes, the famous yet elusive detective. Holmes is ruthlessly and perfectly logical – he takes each statement at its logical meaning. He does not believe you will lie, but he does not accept unstated intentions.

You want Holmes to show you around London, but you also know he hates showing people around London. However, you know a secret about Holmes: he hates cleaning his house. So, you've resorted to bribery — you're willing to clean his house in exchange for compliance. Holmes' first choice would be to get his house cleaned and not show people around London; his second choice would be to get his house cleaned and show people around London.

Said differently, he would rather have his house cleaned and show you around than do neither, but he will not show you around without the reward of getting his house cleaned, nor will he show you around if he has any hope of getting his house cleaned some other way.

- (a) You tell Holmes “If you don’t show me around, then I won’t clean your house.”  
He looks at you and says “It is not yet logical to show you around London.” Why will he not comply? (1-2 sentences)
- (b) You try again: “Forget the first promise,” you say. “If you show me around London, then I will clean your house.”  
Still he says, “It is not yet logical to show you around London.” Why does he still not comply? (1-2 sentences)
- (c) Give a logical sentence (or sentences) which will compel Holmes to show you around London.  
You cannot just **assert** that he will show you around (e.g., you cannot just say “You will show me around London.”) you must give promise(s) to make it the only logical choice to show you around London.  
Additionally, argue that it will be the only logical choice to show you around London. (2-4 sentences)

## 7. Proof

In [Lecture 2](#) (slides 40 - 59) we gave a symbolic proof that  $(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv (\neg a \vee b)$ . In this problem, we will give another proof.

- (a) Our intuition for the proof in class was “the last two pieces of the formula correspond to vacuous truth (when  $a$  is false).” Identify a commonality in the first two pieces of the formula and describe it.  
Note: Your description should be similar in spirit to the one from class, but it’s ok if you don’t have a technical term like ‘vacuous truth’.
- (b) Give another proof of the formula that matches the intuition from part a instead of the intuition from class.  
Read the [symbolic proof guidelines](#) on the assignments page before you start.  
**Hint:** your proof, if it matches your intuition from (a), will be different from the one from class – at least some of the intermediate expressions will be different.
- (c) In class we labeled portions of the proof in purple with high-level descriptions of what they are doing (lecture 2 slide 57, left side). Produce similar labels for your proof. Submit your answer in the form “Steps [X] to [Y]: [label].”  
Note: The goal here is to give intuition for what is happening at a higher level than individual steps.

## 8. Feedback [Extra Credit]

**Answer these questions on the separate gradescope box for this question.**

Please keep track of how much time you spend on this homework and answer the following questions. This can help us calibrate future assignments and future iterations of the course, and can help you identify which areas are most challenging for you.

- How many hours did you spend working on this assignment (excluding any extra credit questions, if applicable)? Report your estimate to the nearest hour.
- Which problem did you spend the most time on?
- Any other feedback for us?