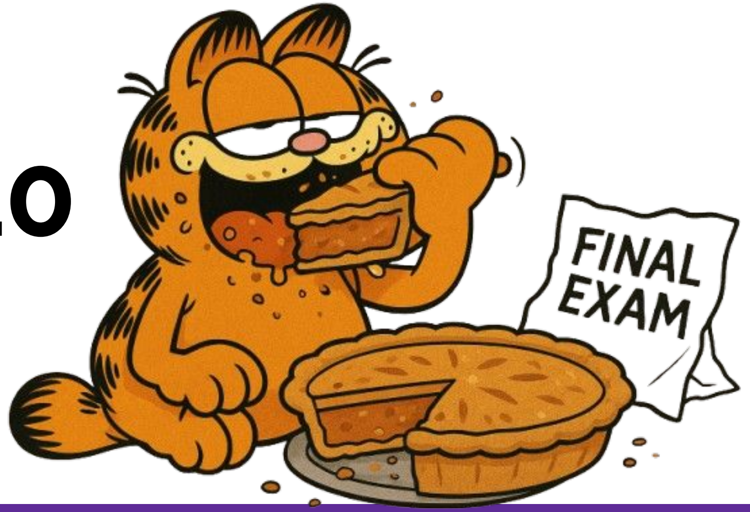
CSE 311 Section 10



Final Review

Announcements & Reminders

- HW8
  - Due **today**@ 11:00
- Final Exam
  - Monday KANE 120 12:30-2:20
  - Bring your Husky ID
- 🎉 **Final Review Session**
  - Come hang out!
  - Friday BAG 131 4:00-5:30pm
- Book one on ones! (linked on Ed)

- **Course Evaluations are out!**
  - Please consider taking 10 minutes to complete both section and course evaluations!

Irregularity

Irregularity Template

Claim: L is an irregular language.

Proof: Suppose, for the sake of contradiction, that L is regular. Then there is a DFA M such that M accepts exactly L.

Let S = [TODO] (S is an infinite set of strings)
Because the DFA is finite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. [TODO] (We don't get to choose x, y, but we can describe them based on that set S we just defined)

Consider the string z = [TODO] (We do get to choose z depending on x, y)

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that xz = [TODO], so $xz \in L$ but yz = [TODO], so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

Therefore, L is an irregular language.

Irregularity Example from Lecture

Claim: $\{0^k 1^k : k \geq 0\}$ is an irregular language.

Proof: Suppose, for the sake of contradiction, that $L = \{0^k 1^k : k \geq 0\}$ is regular. Then there is a DFA M such that M accepts exactly L.

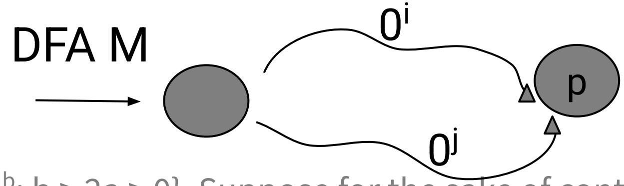
Let $S = \{0^k : k \geq 0\}$
Because the DFA is finite and S is infinite, there are two (different) strings x, y in S such that x and y go to the same state when read by M. Since both are in S, $x = 0^a$ for some integer $a \geq 0$, and $y = 0^b$ for some integer $b \geq 0$, with $a \neq b$.

Consider the string $z = 1^a$.

Since x, y led to the same state and M is deterministic, xz and yz will also lead to the same state q in M. Observe that $xz = 0^a 1^a$, so $xz \in L$ but $yz = 0^b 1^a$, so $yz \notin L$. Since q is can be only one of an accept or reject state, M does not actually recognize L. That's a contradiction!

Therefore, L is an irregular language.

1) Irregularity



Define L, and intro: Let $L = \{0^a1^b : b \geq 2a \geq 0\}$. Suppose for the sake of contradiction that some DFA M recognizes L.

Define S: Consider S = ______. Since S contains infinitely many strings and M has a finite number of states, two distinct strings __, __ ∈ S must end up in the same state p.

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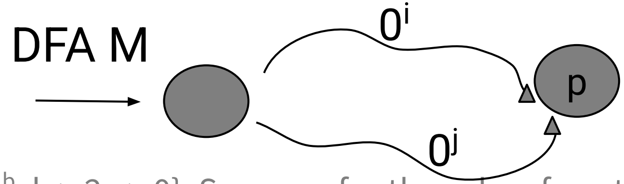

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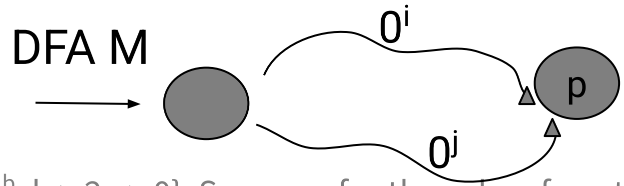

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Append a common suffix string (based on the prefix strings):

We go **by cases**, since we have either i < j or i > j.

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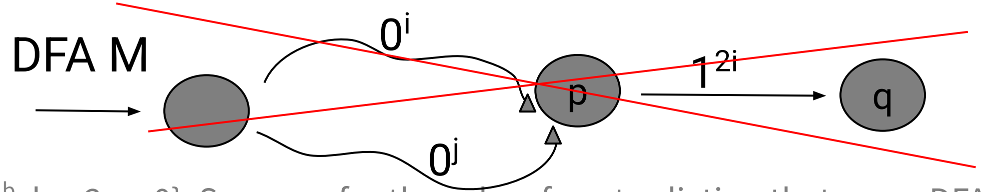

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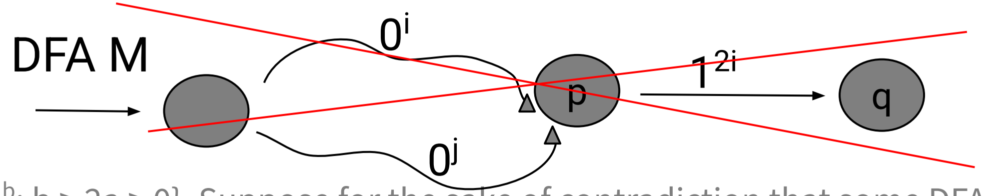

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Argue one string in the language and other isn't (so we have a contradiction!):

Since we appended the same string, 0^i1^{2i} and 0^j1^{2i} end up in the same state q of M, but one is accepted and the other is rejected — contradiction since q can't be both a reject and accept state!

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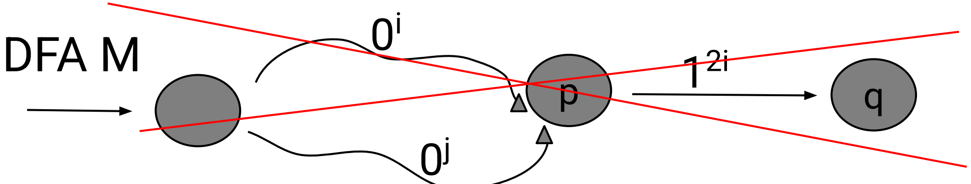

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Argue one string in the language and other isn't (so we have a contradiction!):

$0^i\mathbf{1}^{\mathbf{2i}} \in L$ since $a = i$, $b = 2i$ and $b = 2a$, so $b \geq 2a$.✅

Since we appended the same string, 0^i1^{2i} and 0^j1^{2i} end up in the same state q of M, but one is accepted and the other is rejected — contradiction since q can't be both a reject and accept state!

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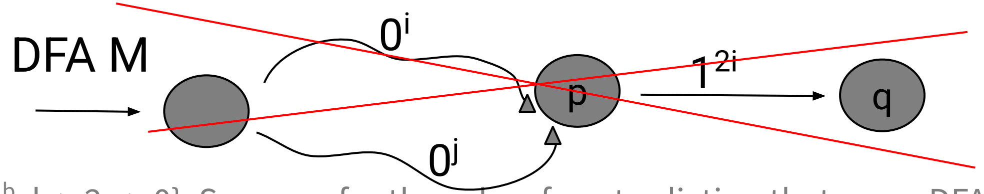

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$0^i\mathbf{1}^{\mathbf{2i}} \in L$ since $a = i$, $b = 2i$ and $b = 2a$, so $b \geq 2a$.✅

$0^j\mathbf{1}^{\mathbf{2i}} \notin L$ since $a = j$ and $b = 2i$, but $i < j$ so we have $2i < 2j$, meaning that $b < 2a$. ❌

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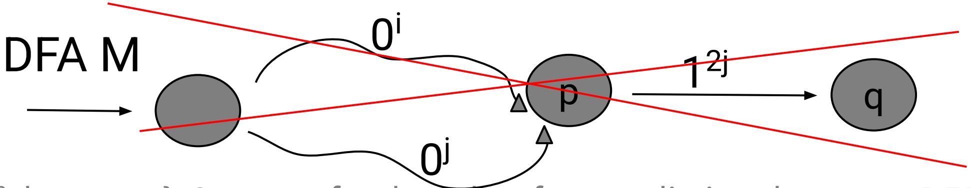

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Case 1: Completed!

Case 2: Suppose $j < i$. Consider appending 1^{2j} to both 0^i and 0^j.

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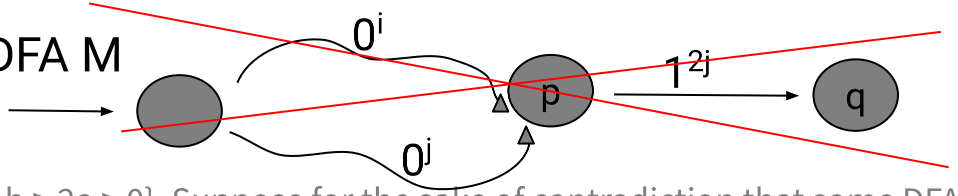

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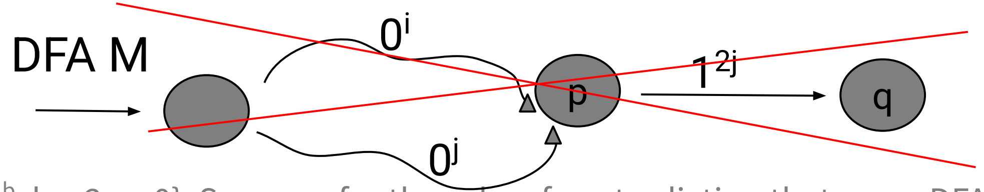

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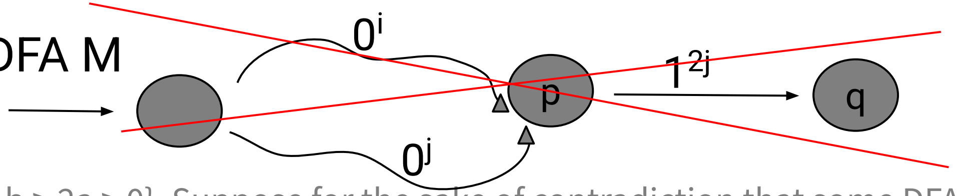

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$0^j1^{2j} \in L$ since $a = j$ and $b = 2j$ and $b = 2a$, so $b \geq 2a$. ✅

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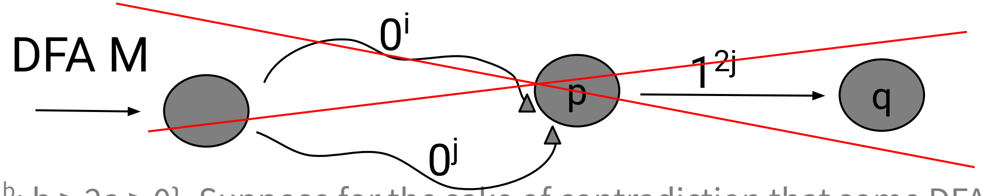

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Append a common suffix string (based on the prefix strings):

We go **by cases**, since we have either $i < j$ or $i > j$.

Case 1: Completed!

Case 2: Completed!

We have a contradiction in all cases!

Conclusion: We don't have a DFA that recognizes L, so L is not regular.

1) Irregularity Tips

- Many correct choices for the infinite set S of partial prefix strings.
  - S doesn't need to account for all partial strings; it can be a subset. It does need to be **infinite**.
- You **do not** get to choose which two prefix strings end up at the same intermediate state of the DFA.
  - But you do know **they are in S** and that they are **distinct**.
- You **do** get to choose the common suffix string to append based on the two prefix strings that ended up in the same state. Choose wisely, figure out what the DFA needs to "count" :D
- Template for irregularity
  - Choose S, common string to append (based on prefix string structure), argue why one string in the language and other isn't

Strong Induction Review

Task 7: Strong Induction

Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

$$
\begin{aligned}
a_1 &= 1 \\
a_2 &= 2 \\
a_3 &= 5 \\
a_n &= 3a_{n-1} + 4a_{n-2} + a_{n-3} && \text{for } n \geqslant 4
\end{aligned}
$$

Prove that $a_n \geqslant 4^{n-2}$ for all integers $n \geqslant 1$.

Strong Induction

Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

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Prove that $a_n \geqslant 4^{n-2}$ for all integers $n \geqslant 1$.

Base cases:

Conclusion: P(n) holds for all n ≥ 1.

Strong Induction

Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

$$a_1 = 1$$
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$$a_n = 3a_{n-1} + 4a_{n-2} + a_{n-3} \qquad \text{for } n \geqslant 4$$

Prove that $a_n \geqslant 4^{n-2}$ for all integers $n \geqslant 1$.

Base cases:

$n = 1$: $a_1 = 1 \geq 4^{-1} = 4^{1-2}$

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Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

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n = 3: $a_3 = 5 \geq 4^{1} = 4^{3-2}$

Inductive hypothesis Suppose P(j) holds for all 1 < j ≤ k (k ≥ 3), i.e. a□ $\geq 4^{j-2}$ for *j = k, k−1, k−2…*

Conclusion: P(n) holds for all n ≥ 1.

Strong Induction

Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

$$a_1 = 1$$
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Inductive step – show P(k + 1)

Conclusion: P(n) holds for all n ≥ 1.

Strong Induction

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$a_{□+1} =$

$\geq 4^{k-1}$

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$a_{k+1} = 3\,a_k + 4\,a_{k-1} + a_{k-2}$ (definition; $k + 1 \geq 4$)

$\geq 4^{k-1}$

Conclusion: P(n) holds for all $n \geq 1$.

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Inductive hypothesis Suppose P(j) holds for all $1 < j \leq k$ ($k \geq 3$), i.e. $a_{\square} \geq 4^{j-2}$ for *$j = k, k-1, k-2\ldots$*

Inductive step – show P(k + 1)

$a_{\square+1} = 3\, a_{\square} + 4\, a_{\square-1} + a_{\square-2}$ (definition; $k + 1 \geq 4$)

$\geq 3 \cdot 4^{k-2} + 4 \cdot 4^{k-3} + 4^{k-4}$ (I.H.)

$\geq 4^{k-1}$

Conclusion: P(n) holds for all $n \geq 1$.

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$a_{k+1} = 3\,a_k + 4\,a_{k-1} + a_{k-2}$ (definition; $k + 1 \geq 4$)

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Prove that $a_n \geqslant 4^{n-2}$ for all integers $n \geqslant 1$.

Base cases:

$n = 1$: $a_1 = 1 \geq 4^{-1} = 4^{1-2}$

$n = 2$: $a_2 = 2 \geq 4^{0} = 4^{2-2}$

$n = 3$: $a_3 = 5 \geq 4^{1} = 4^{3-2}$

Inductive hypothesis Suppose P(j) holds for all $1 < j \leq k$ ($k \geq 3$), i.e. $a_{\square} \geq 4^{j-2}$ for $j = k, k-1, k-2\ldots$

Inductive step – show P(k + 1)

$a_{\square+1} = 3\,a_{\square} + 4\,a_{\square-1} + a_{\square-2}$ (definition; $k + 1 \geq 4$)

$\geq 3 \cdot 4^{k-2} + 4 \cdot 4^{k-3} + 4^{k-4}$ (I.H.)

$= 3 \cdot 4^{k-2} + 4^{k-2} + 4^{k-4}$

$= 4 \cdot 4^{k-2} + 4^{k-4}$

$\geq 4^{k-1}$

Conclusion: P(n) holds for all $n \geq 1$.

Strong Induction

Define a sequence of positive integers a_n with $n \geqslant 1$ as follows:

$$a_1 = 1$$
$$a_2 = 2$$
$$a_3 = 5$$
$$a_n = 3a_{n-1} + 4a_{n-2} + a_{n-3} \qquad \text{for } n \geqslant 4$$

Prove that $a_n \geqslant 4^{n-2}$ for all integers $n \geqslant 1$.

Base cases:

n = 1: $a_1 = 1 \geq 4^{-1} = 4^{1-2}$

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$= 4^{k-1} + 4^{k-4}$

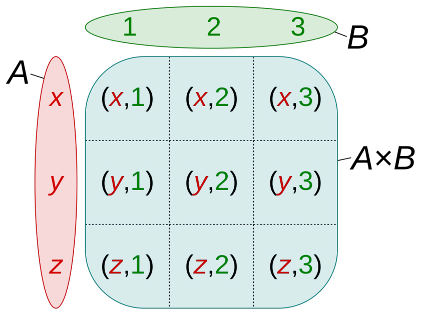
$\geq 4^{k-1}$

Conclusion: P(n) holds for all $n \geq 1$.

Set Theory review!

Set Operators

- Subset: $A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$
- Equality: $A = B \equiv \forall x(x \in A \leftrightarrow x \in B) \equiv A \subseteq B \wedge B \subseteq A$
- Union: $A \cup B = \{x : x \in A \vee x \in B\}$
- Intersection: $A \cap B = \{x : x \in A \wedge x \in B\}$
- Complement: $\overline{A} = \{x : x \notin A\}$
- Difference: $A \backslash \mathrm{B} = \{x : x \in A \wedge x \notin B\}$
- Cartesian Product: $A \times B = \{(a, b) : a \in A \wedge b \in B\}$



Set Theory: Casework + Powersets

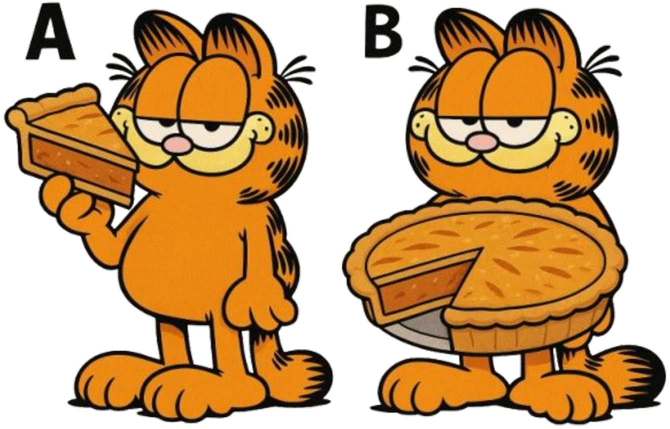
Powerful Sets (Task 9)

Show that for any set X and any set A with $A \in \mathcal{P}(X)$, there exists a set $B \in \mathcal{P}(X)$ such that $A \cap B = \varnothing$ and $A \cup B = X$.

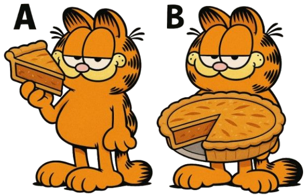
Powerful Sets (Task 9)

Show that for any set X and any set A with $A \in \mathcal{P}(X)$, there exists a set $B \in \mathcal{P}(X)$ such that $A \cap B = \varnothing$ and $A \cup B = X$.

Intuition: Imagine X as an **entire pie** from which **you've removed one slice** called A. Whatever remains—called **B** which is **X \ A**.
By construction, A and B **share no crumbs (A ∩ B = ∅)**, yet **together** they still fill the whole pie **(A ∪ B = X)**.



Powerful Sets (Task 9)



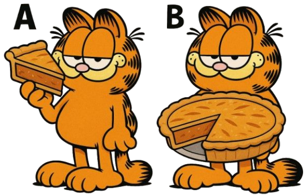
Show that for any set X and any set A with $A \in \mathcal{P}(X)$, there exists a set $B \in \mathcal{P}(X)$ such that $A \cap B = \varnothing$ and $A \cup B = X$.

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$)
2. $A \cap B = \varnothing$ (disjoint)
3. $A \cup B = X$ (covers X)

Powerful Sets (Task 9)



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Let B = X \ A.

1. Goal: show $B \in \mathscr{P}(X)$.

*We will provide a concise solution here, full solution on handout solutions

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$)
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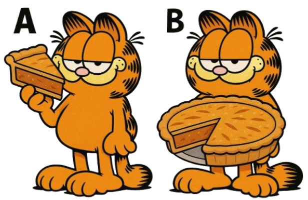

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Let $B = X \setminus A$.

1. $B \in \mathscr{P}(X)$.
   - Let x be an arbitrary object. Suppose $x \in B$.

Goals

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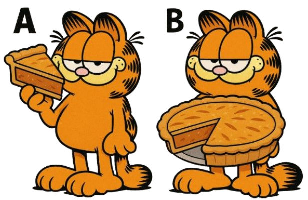

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   - Let x be an arbitrary object. Suppose $x \in B$.
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Goals

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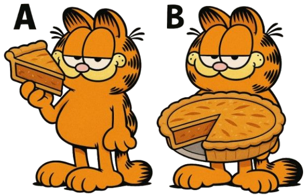

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   - Since $x \in X$ and was arbitrary, $B \subseteq X$ and thus $B \in \mathscr{P}(X)$.

Goals

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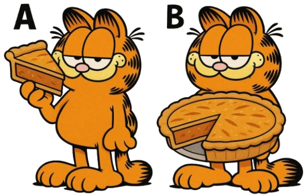

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Goals

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1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$) ✅
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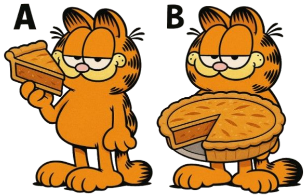

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Let B = X \ A.

1. B ∈ $\mathscr{P}$(X).
   - Let x be an arbitrary object. Suppose x ∈ B.
   - Since B = X \ A, by definition of "\" we have x ∈ X and x ∉ A.
   - Since x ∈ X and was arbitrary, B ⊆ X and thus B ∈ $\mathscr{P}$(X).

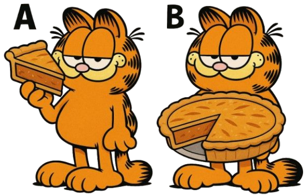
2. A ∩ B = ∅

Goals

-----

1. B ∈ $\mathscr{P}$(X) (i.e. B ⊆ X) ✅
2. A ∩ B = ∅ (disjoint)
3. A ∪ B = X (covers X)

Powerful Sets (Task 9)



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2. $A \cap B = \varnothing$
   - Let x be an arbitrary object. Suppose $x \in B$.

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$) ✅
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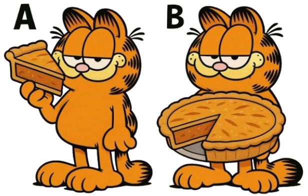
Powerful Sets (Task 9)



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2. $A \cap B = \varnothing$
   - Let x be an arbitrary object. Suppose $x \in B$.
   - Since $B = X \setminus A$, $x \in X$ and $x \notin A$. (again by definition of "\").

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$) ✅
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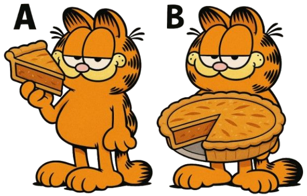

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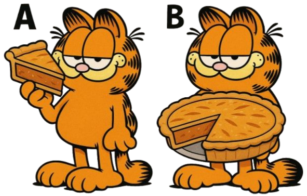
2. $A \cap B = \varnothing$
   - Let x be an arbitrary object. Suppose $x \in B$.
   - Since $B = X \setminus A$, $x \in X$ and $x \notin A$. (again by definition of "\").
   - Thus no element lies in both sets, so $A \cap B = \varnothing$.

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$) ✅
2. $A \cap B = \varnothing$ (disjoint) ✅
3. $A \cup B = X$ (covers X)

Powerful Sets (Task 9)



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3. A ∪ B = X
   (a) A ∪ B ⊆ X

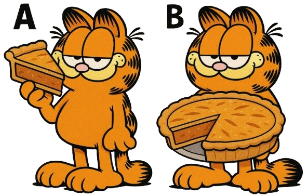
   (b) X ⊆ A ∪ B

Goals

-----

1. B ∈ 𝒫(X) (i.e. B ⊆ X) ✅
2. A ∩ B = ∅ (disjoint) ✅
3. A ∪ B = X (covers X)

Powerful Sets (Task 9)



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   (a) A ∪ B ⊆ X
      – Let x be an arbitrary object. Suppose x ∈ A.

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-----

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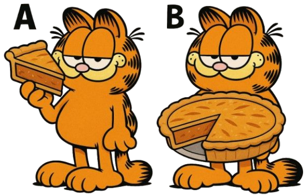

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3. A ∪ B = X
   (a) A ∪ B ⊆ X
   – Let x be an arbitrary object. Suppose x ∈ A.
   – Then x ∈ X because A ⊆ X.

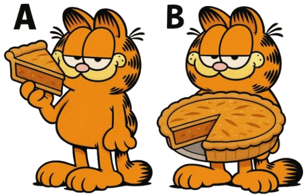
   (b) X ⊆ A ∪ B

Goals

-----

1. B ∈ 𝒫(X)   (i.e. B ⊆ X) ✅
2. A ∩ B = ∅   (disjoint) ✅
3. A ∪ B = X   (covers X)

Powerful Sets (Task 9)



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3. A ∪ B = X
   (a) A ∪ B ⊆ X
      – Let x be an arbitrary object. Suppose x ∈ A.
      – Then x ∈ X because A ⊆ X.
      – If x ∈ B, then x ∈ X by part 1.

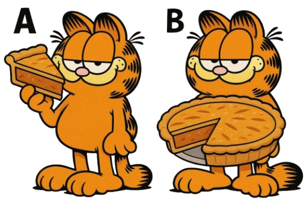
   (b) X ⊆ A ∪ B

Goals

-----

1. B ∈ 𝒫(X)   (i.e. B ⊆ X) ✅
2. A ⋂ B = ∅   (disjoint) ✅
3. A ∪ B = X   (covers X)

Powerful Sets (Task 9)



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      – If x ∈ B, then x ∈ X by part 1.
      – Therefore x ∈ X in either case.
   (b) X ⊆ A ∪ B

Goals

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1. B ∈ 𝒫(X) (i.e. B ⊆ X) ✅
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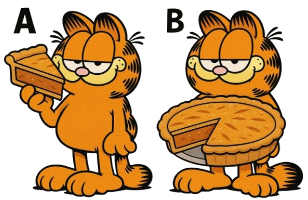

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   - Therefore x ∈ X in either case.

   (b) X ⊆ A ∪ B
   - Let x be an arbitrary object. Suppose x ∈ X.

Goals

-----

1. B ∈ $\mathscr{P}$(X)   (i.e. B ⊆ X) ✅
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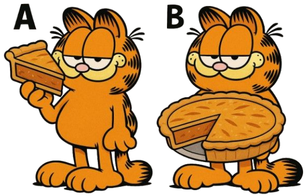
Powerful Sets (Task 9)



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   (b) X ⊆ A ∪ B
   - Let x be an arbitrary object. Suppose x ∈ X.
   - If x ∈ A, then (x ∈ A ∪ B) by the definition of union.

Goals

-----

1. B ∈ $\mathscr{P}$(X) (i.e. B ⊆ X) ✅
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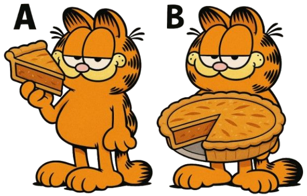
Powerful Sets (Task 9)



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   – If x ∈ A, then (x ∈ A ∪ B) by the definition of union.
   – If x ∉ A, then x ∈ X \ A = B, so x ∈ A ∪ B.

Goals

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1. B ∈ 𝒫(X) (i.e. B ⊆ X) ✅
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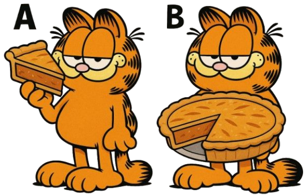

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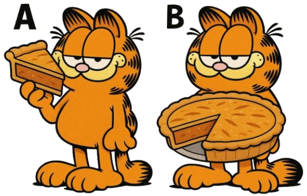
3. A ∪ B = X
   (a) A ∪ B ⊆ X
   – Let x be an arbitrary object. Suppose x ∈ A.
   – Then x ∈ X because A ⊆ X.
   – If x ∈ B, then x ∈ X by part 1.
   – Therefore x ∈ X in either case.
   (b) X ⊆ A ∪ B
   – Let x be an arbitrary object. Suppose x ∈ X.
   – If x ∈ A, then (x ∈ A ∪ B) by the definition of union.
   – If x ∉ A, then x ∈ X \ A = B, so x ∈ A ∪ B.
   • Both inclusions hold, hence A ∪ B = X.

Goals

-----

1. B ∈ 𝒫(X) (i.e. B ⊆ X) ✅
2. A ∩ B = ∅ (disjoint) ✅
3. A ∪ B = X (covers X)

Powerful Sets (Task 9)



Show that for any set X and any set A with $A \in \mathcal{P}(X)$, there exists a set $B \in \mathcal{P}(X)$ such that $A \cap B = \varnothing$ and $A \cup B = X$.

3. $A \cup B = X$
   (a) $A \cup B \subseteq X$
   - Let x be an arbitrary object. Suppose $x \in A$.
   - Then $x \in X$ because $A \subseteq X$.
   - If $x \in B$, then $x \in X$ by part 1.
   - Therefore $x \in X$ in either case.

   (b) $X \subseteq A \cup B$
   - Let x be an arbitrary object. Suppose $x \in X$.
   - If $x \in A$, then $(x \in A \cup B)$ by the definition of union.
   - If $x \notin A$, then $x \in X \setminus A = B$, so $x \in A \cup B$.
   - Both inclusions hold, hence $A \cup B = X$.

Conclusion

For $B = X \setminus A$, we have shown that $B \in \mathscr{P}(X)$, $A \cap B = \varnothing$, $A \cup B = X$.

Goals

-----

1. $B \in \mathscr{P}(X)$ (i.e. $B \subseteq X$) ✅
2. $A \cap B = \varnothing$ (disjoint) ✅
3. $A \cup B = X$ (covers X) ✅

DFA/Regex/CFG review

Task 5a

Construct a regular expression that represents binary strings where no occurence of 11 is followed by a 0.

Task 5a

Construct a regular expression that represents binary strings where no occurence of 11 is followed by a 0.

Write out some examples:

Accepted Strings	Rejected Strings
ε, 0, 1, 00, 01	0110
01011	101011**0**
101010111	01011**0**10

Task 5a

Construct a regular expression that represents binary strings where no occurence of 11 is followed by a 0.

Observations: The string can be split into the first part where there are no 11's and the string can end in any number of 1's

Accepted Strings	Rejected Strings
ε, 0, 1, 00, 01	0110
01011	101011**0**
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Task 5a

Construct a regular expression that represents binary strings where no occurence of 11 is followed by a 0.

Observations: The string can be split into the first part where there are no 11's and the string can end in any number of 1's

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Task 5a

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Observations: The string can be split into the first part where there are no 11's and the string can end in any number of 1's

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Strings without the substring "11"

$(0^* \cup (10)^*)^*$ or $(0^*(10)^*)^*$

Task 5a

Construct a regular expression that represents binary strings where no occurence of 11 is followed by a 0.

Observations: The string can be split into the first part where there are no 11's and the string can end in any number of 1's

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ε, 0, 1, 00, 01	0110
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Strings without the substring "11"

(0* ∪ (10)*)* or (0*(10)*))*

Strings without the substring "11" ends with any number of 1's

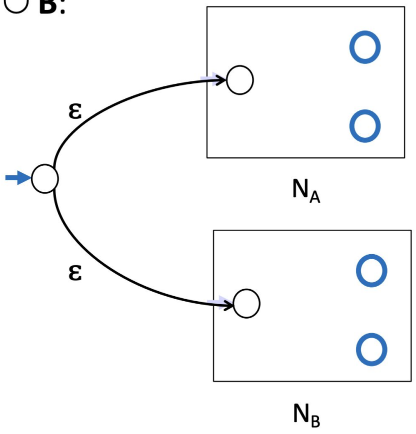
(0* ∪ (10)*)*1* or (0*(10)*))*1*

Task 5b

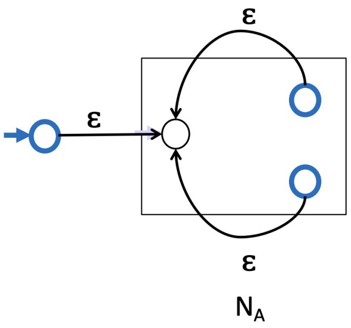
Convert the regular expression "$1(0 \cup 11)^*$" to an NFA using the algorithm from lecture. You may skip adding ε-transitions for concatenation if they are obviously unnecessary, but otherwise, you should follow the construction from lecture.

Regex to NFA Conversion!

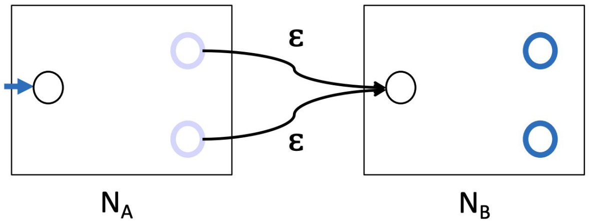
Case A ∪ B:



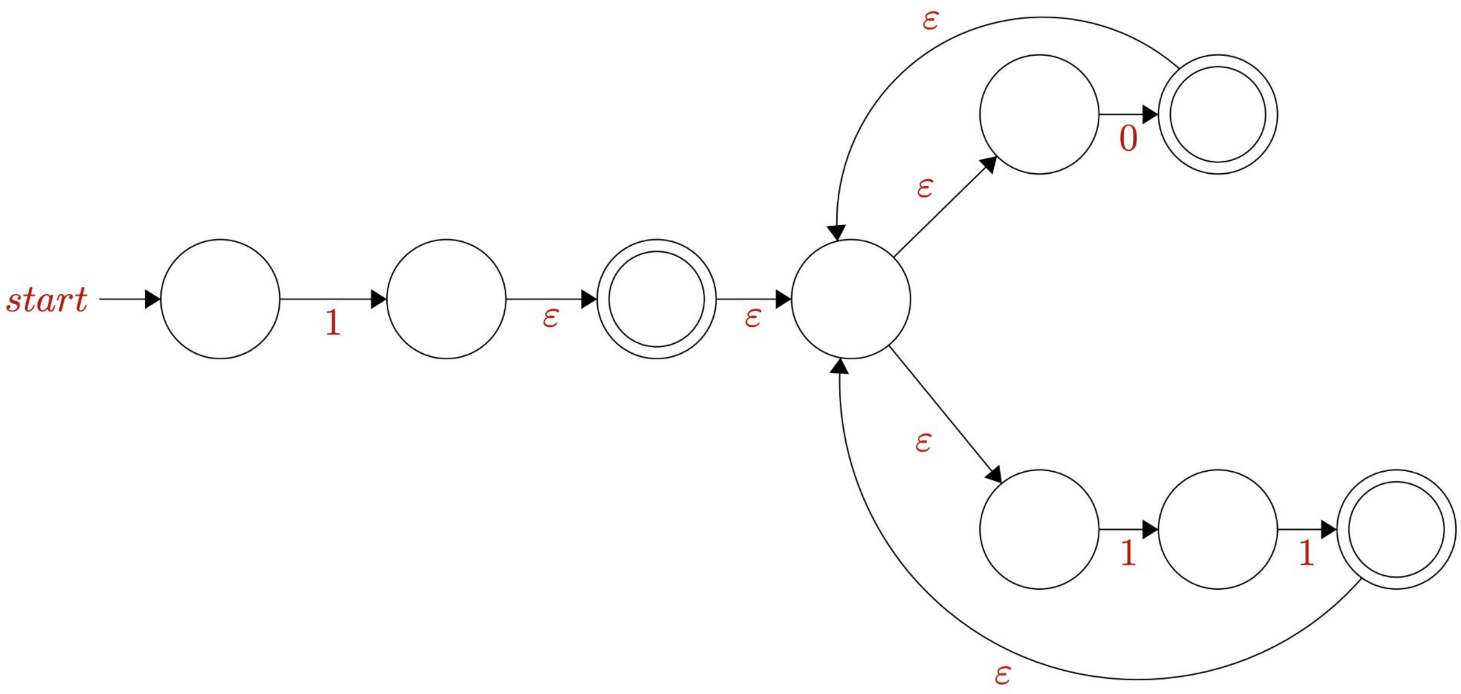
Case A*



Case AB:



Regex to NFA Conversion!



Task 5c

Construct a CFG that represents the following language: $\{1^x 2^y 3^y 4^x : x, y \geqslant 0\}$.

Whiteboard problem!

Task 5c

Construct a CFG that represents the following language: $\{1^x 2^y 3^y 4^x : x, y \geqslant 0\}$.

$$\mathbf{S} \to 1\mathbf{S}4 \mid \mathbf{T}$$
$$\mathbf{T} \to 2\mathbf{T}3 \mid \varepsilon$$

Set Theory 2

Task 8

1. Prove or disprove: For all sets A, B, C if $A \cap C = B \cap C$ then $A = B$.

Task 8

1. Prove or disprove: For all sets A, B, C if $A \cap C = B \cap C$ then $A = B$.

This claim is false. We prove via counterexample:
A: {3,2}
B: {4,2}
C: {2}

Task 8

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Although {2} = {2}, {4,2} ≠ {3,2} (A ≠ B)

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Although {2} = {2}, {4,2} ≠ {3,2} (A ≠ B)

Simpler example:
A = {1}
B = {2}
C = {}

Task 8

1. Prove or disprove: For all sets A, B, C if $A \cap C = B \cap C$ then $A = B$.

This claim is false. We prove via counterexample:
A: {3,2}
B: {4,2}
C: {2}

Although {2} = {2}, {4,2} ≠ {3,2} (A ≠ B)

Simpler example:
A = {1}
B = {2}
C = {}
Although {} = {}, {1} ≠ {2} (A ≠ B)

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ then $A = B$.

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B = {2}
C = {1,2}

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ then $A = B$.

This claim is false. We prove via counterexample:
A = {1}
B = {2}
C = {1,2}

Although
A U C = {1,2}
B U C = {1,2}

{1} ≠ {2} (A ≠ B)

Task 8

Prove or disprove: Let A, B, C be arbitrary sets. For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

Can we use the meta-theorem template?

Task 8

Prove or disprove: Let A, B, C be arbitrary sets. For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

Hint: if you know x $\in$ A$\cap$C and what do you know now? What about x $\in$ A$\cup$C?

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Suppose that A ∪ C = B ∪ C and A∩C = B∩C.

⊆: **We aim to show that A ⊆ B**. Let x ∈ A be arbitrary.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\subseteq$: **We aim to show that $A \subseteq B$**. Let $x \in A$ be arbitrary.

Case 1: $x \in A$ and $x \in C$.

Case 2: $x \in A$ and $x \notin C$.

Thus in all cases $A \subseteq B$.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that A ∪ C = B ∪ C and A∩C = B∩C.

⊆: **We aim to show that A ⊆ B**. Let x ∈ A be arbitrary.

Case 1: x ∈ A and x ∈ C.

We have **x ∈ B**.

Case 2: x ∈ A and x ∉ C.

we have **x ∈ B**.

Since x was arbitrary, A ⊆ B.

Thus in all cases A ⊆ B.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\subseteq$: **We aim to show that $A \subseteq B$**. Let $x \in A$ be arbitrary.

Case 1: $x \in A$ and $x \in C$.

Then by definition of intersection, $x \in A \cap C$.

So $x \in B$.

Case 2: $x \in A$ and $x \notin C$.

Since $x \in A$, by definition of union, $x \in A \cup C$.

we have $x \in B$.

Since x was arbitrary, $A \subseteq B$.

Thus in all cases $A \subseteq B$.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

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Case 1: $x \in A$ and $x \in C$.

Then by definition of intersection, $x \in A \cap C$.

Since $A \cap C = B \cap C$, $x \in B \cap C$.

So $x \in B$.

Case 2: $x \in A$ and $x \notin C$.

Since $x \in A$, by definition of union, $x \in A \cup C$.

Since $A \cup C = B \cup C$, $x \in B \cup C$.

But since $x \notin C$, we have $x \in B$.

Since x was arbitrary, $A \subseteq B$.

Thus in all cases $A \subseteq B$.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\subseteq$: **We aim to show that $A \subseteq B$**. Let $x \in A$ be arbitrary.

Case 1: $x \in A$ and $x \in C$.

Then by definition of intersection, $x \in A \cap C$.

Since $A \cap C = B \cap C$, $x \in B \cap C$.

Then by definition of intersection, $x \in B$ and $x \in C$.

So $x \in B$.

Case 2: $x \in A$ and $x \notin C$.

Since $x \in A$, by definition of union, $x \in A \cup C$.

Since $A \cup C = B \cup C$, $x \in B \cup C$.

Then by definition of union, $x \in B$ or $x \in C$.

But since $x \notin C$, we have $x \in B$.

Since x was arbitrary, $A \subseteq B$.

Thus in all cases $A \subseteq B$.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\subseteq$: **We aim to show that $A \subseteq B$**. Let $x \in A$ be arbitrary.

…

Thus in all cases $A \subseteq B$.

Are we done???

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\subseteq$: **We aim to show that $A \subseteq B$**. Let $x \in A$ be arbitrary.

…

Thus in all cases $A \subseteq B$.

Are we done???

Haha…no, show the other direction!

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\supseteq$: We aim to show that $B \subseteq A$. Let $x \in B$ be arbitrary.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\supseteq$: We aim to show that $B \subseteq A$. Let $x \in B$ be arbitrary.

Case 1: $x \in B$ and $x \in C$.

Case 2: $x \in B$ and $x \notin C$.

Since x was arbitrary, $B \subseteq A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

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Case 1: $x \in B$ and $x \in C$.

$x \in A$.

Case 2: $x \in B$ and $x \notin C$.

$x \in A$.

Thus, in all cases **$x \in A$.**

Since x was arbitrary, $B \subseteq A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

Thus we have shown that $A \subseteq B$ and $B \subseteq A$, so $A = B$, as desired.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\supseteq$: We aim to show that $B \subseteq A$. Let $x \in B$ be arbitrary.

Case 1: $x \in B$ and $x \in C$.

Then by definition of intersection, $x \in B \cap C$

Thus, $x \in A$.

Case 2: $x \in B$ and $x \notin C$.

Then by definition of union, $x \in B \cup C$

As $x \notin C$, $x \in A$.

Thus, in all cases **$x \in A$.**

Since x was arbitrary, $B \subseteq A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

Thus we have shown that $A \subseteq B$ and $B \subseteq A$, so $A = B$, as desired.

Task 8

Prove or disprove: For all sets A, B, C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A = B$.

This claim is true.

Let sets A, B, C be arbitrary, and suppose that $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

$\supseteq$: We aim to show that $B \subseteq A$. Let $x \in B$ be arbitrary.

Case 1: $x \in B$ and $x \in C$.

Then by definition of intersection, $x \in B \cap C$

Since **$A \cap C = B \cap C$,** $x \in$ **$A \cap C$**.

Thus, **$x \in A$**.

Case 2: $x \in B$ and $x \notin C$.

Then by definition of union, $x \in B \cup C$

Since **$A \cup C = B \cup C$**, $x \in$ **$A \cup C$** .

As $x \notin C$, **$x \in A$**.

Thus, in all cases **$x \in A$.**

Since x was arbitrary, $B \subseteq A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

Thus we have shown that $A \subseteq B$ and $B \subseteq A$, so $A = B$, as desired.

You made it! Thank you all for a wonderful quarter!

We are always a resource to you!
Don't hesitate to reach out!