CSE 311 Section 7

Anxiety
garfield

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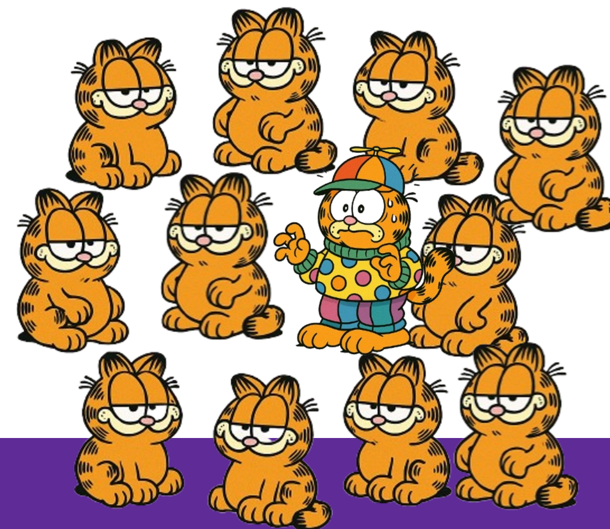


Set Theory & Structural Induction!

Announcements & Reminders

- Congrats on finishing the midterm!
    - Please don't discuss as not everyone has taken it :)

- Homework 6 due Wednesday @ 11:00pm
- **Book One-on-Ones via the link on the message board!**

Sets: Quick Review

Sets

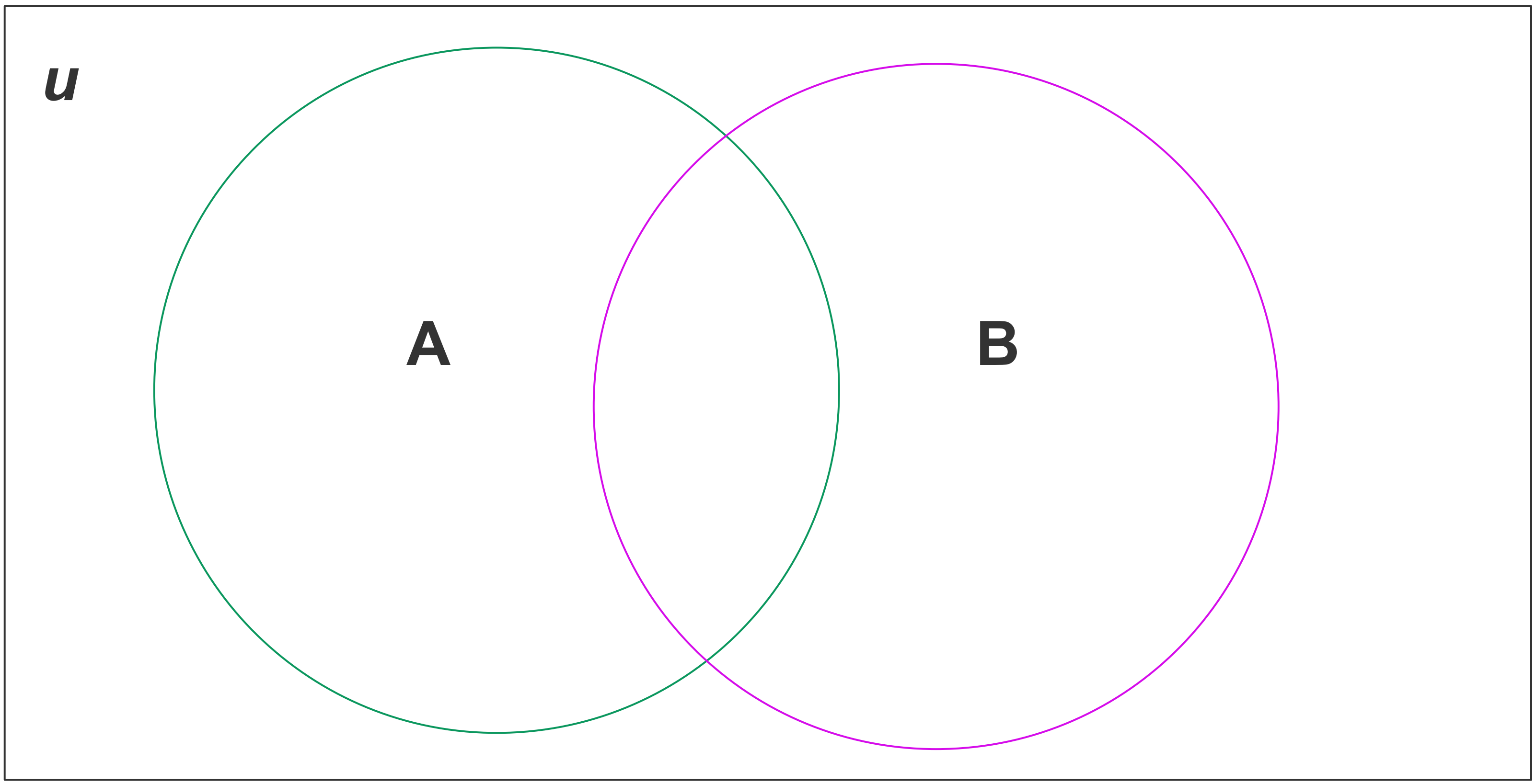
- A set is an **unordered** group of **distinct** elements
  - Set variable names are capital letters, with lower-case letters for elements

- Set Notation:
  - $a \in A$: "a is in A" or "a is an element of A"
  - $A \subseteq B$: "A is a subset of B", every element of A is also in B
  - $\emptyset$: "empty set", a unique set containing no elements
  - $\mathcal{P}(A)$: "power set of A", the set of all subsets of A including the empty set and A itself

Set Operators

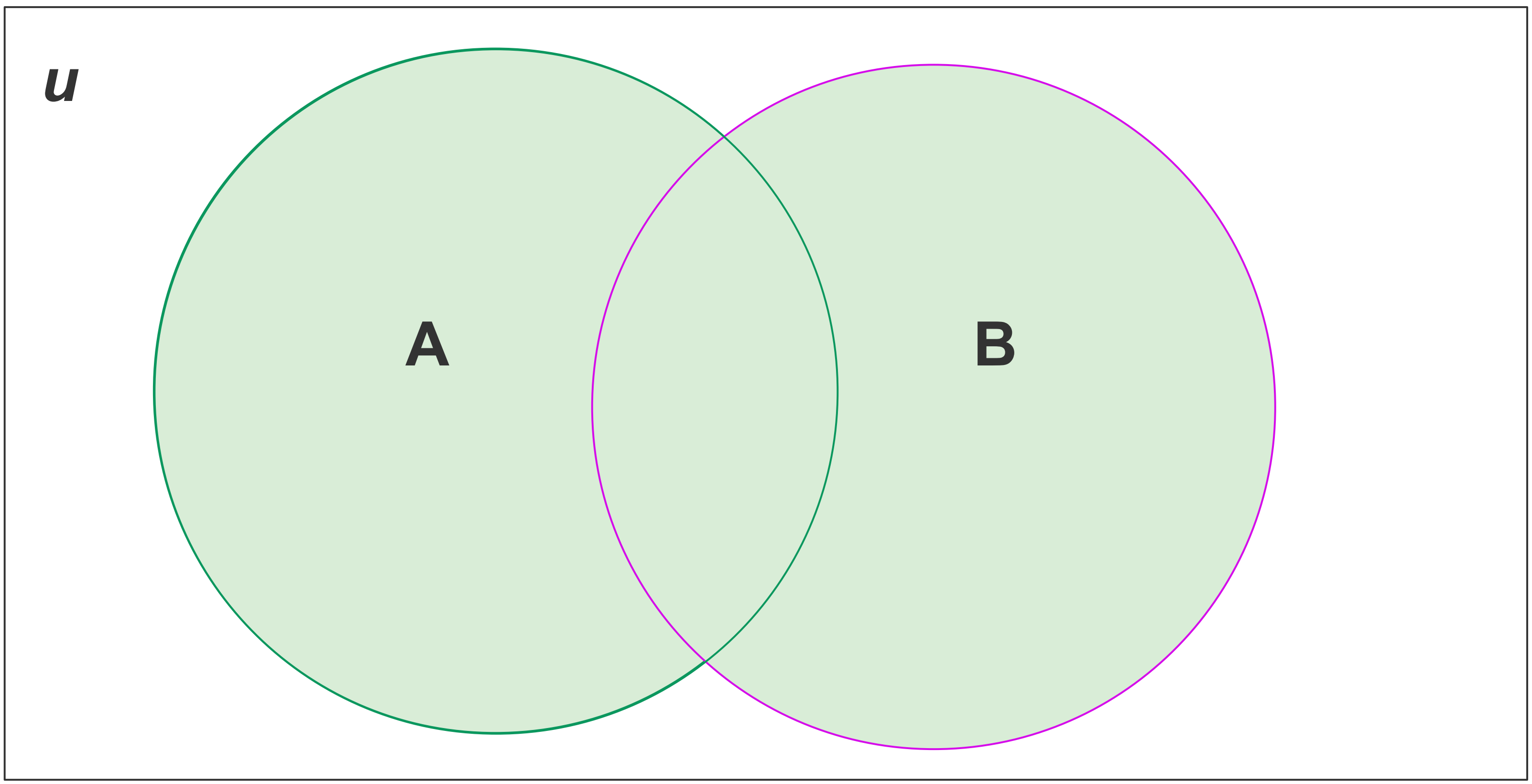
- Subset: $A \subseteq B \equiv \forall x(x \in A \rightarrow x \in B)$
- Equality: $A = B \equiv \forall x(x \in A \leftrightarrow x \in B) \equiv A \subseteq B \wedge B \subseteq A$
- Union: $A \cup B = \{x : x \in A \vee x \in B\}$
- Intersection: $A \cap B = \{x : x \in A \wedge x \in B\}$
- Complement: $\overline{A} = \{x : x \notin A\}$
- Difference: $A \backslash \mathrm{B} = \{x : x \in A \wedge x \notin B\}$
- Cartesian Product: $A \times B = \{(a, b) : a \in A \wedge b \in B\}$

Understand Sets Visually!



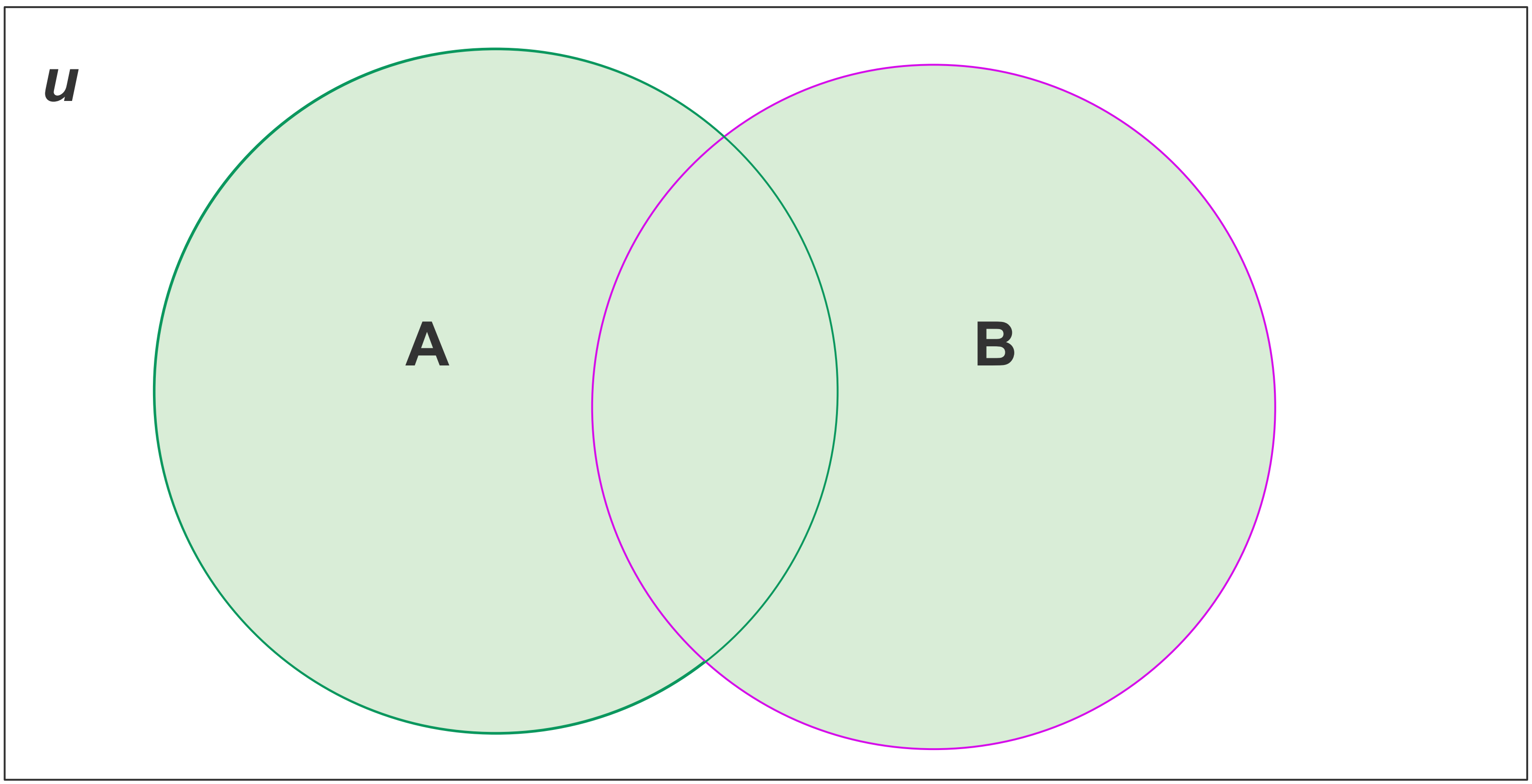
Understand Sets Visually!

What Set Operation is this?



Understand Sets Visually!

What Set Operation is this?
Union: A U B



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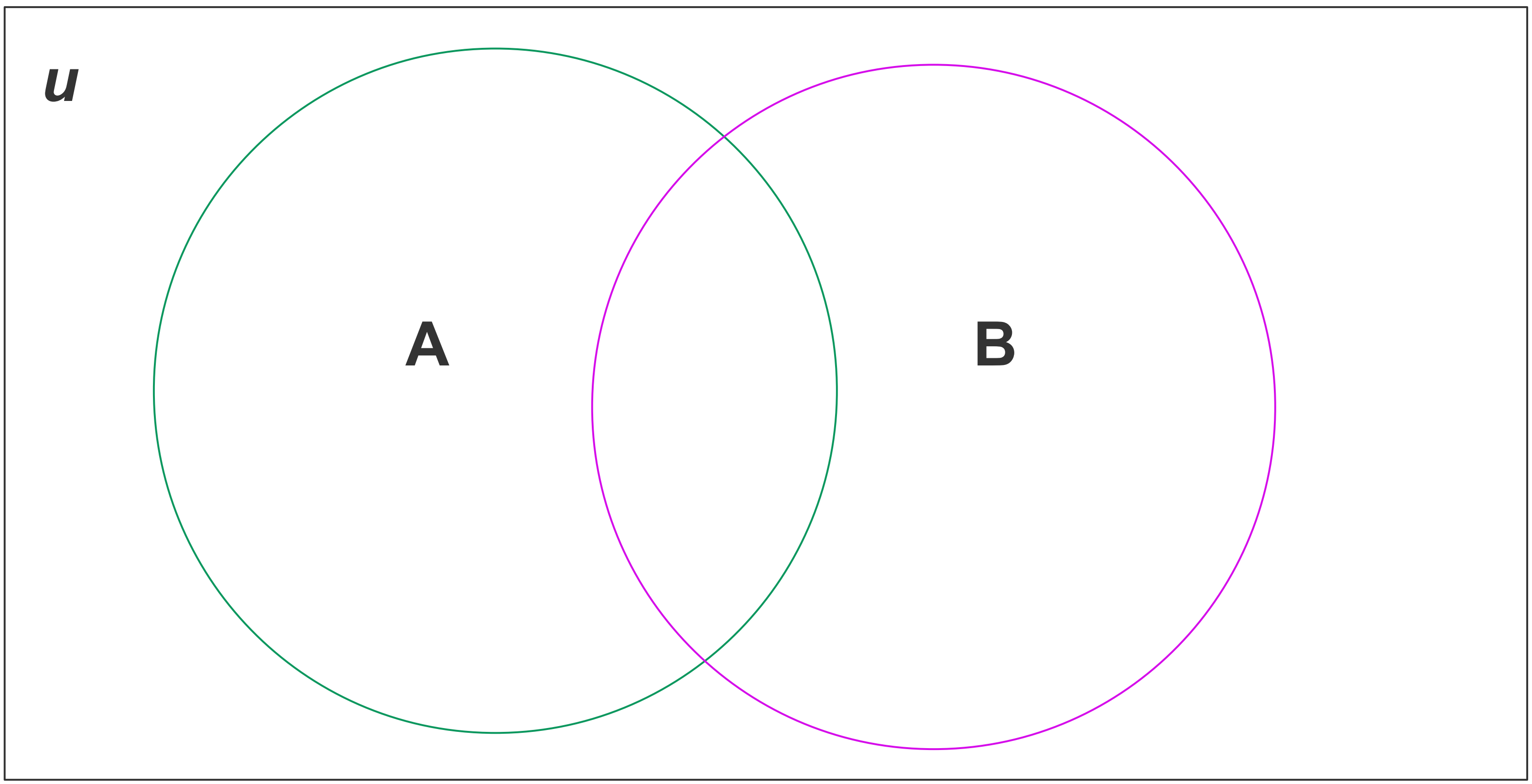

Understand Sets Visually!

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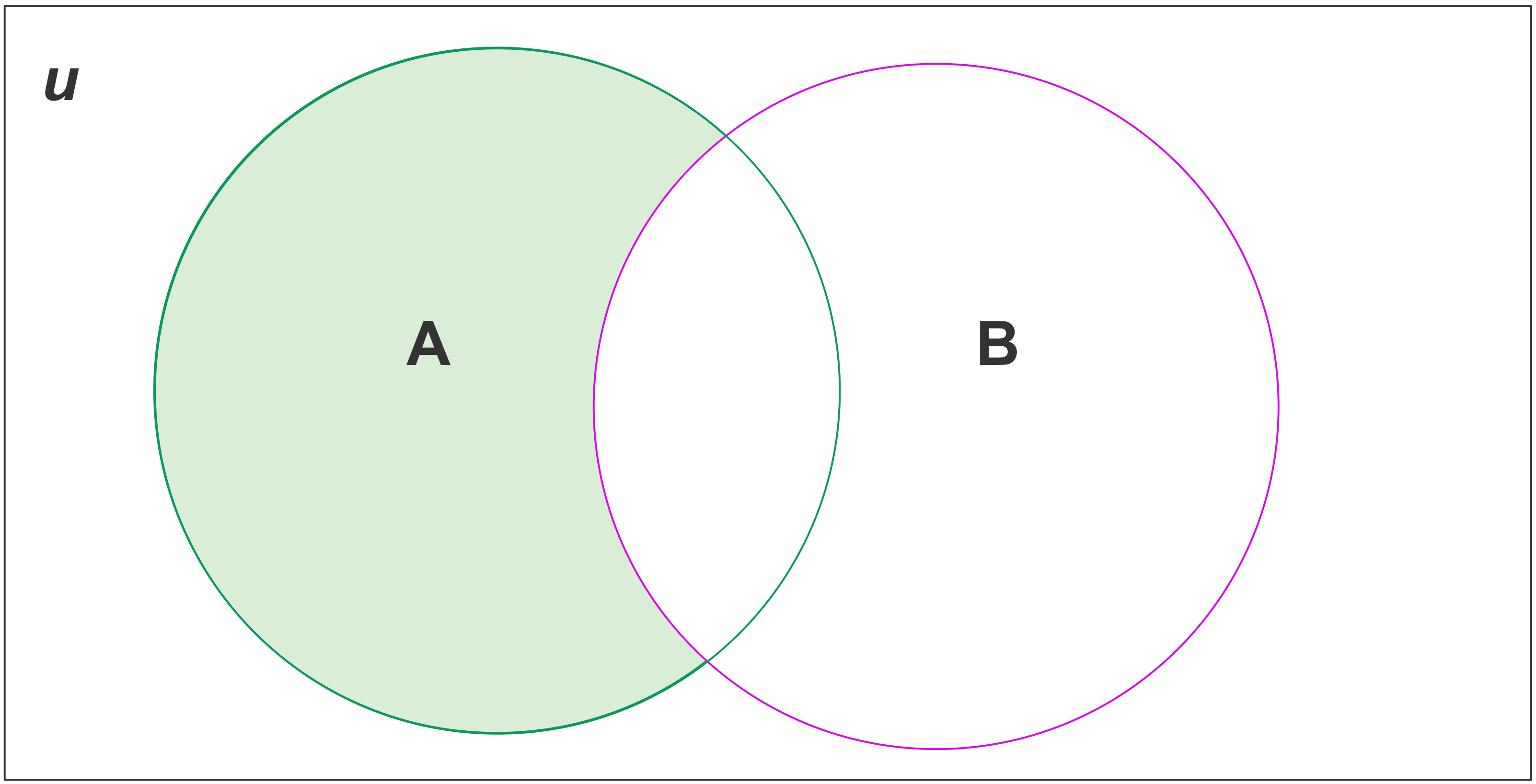
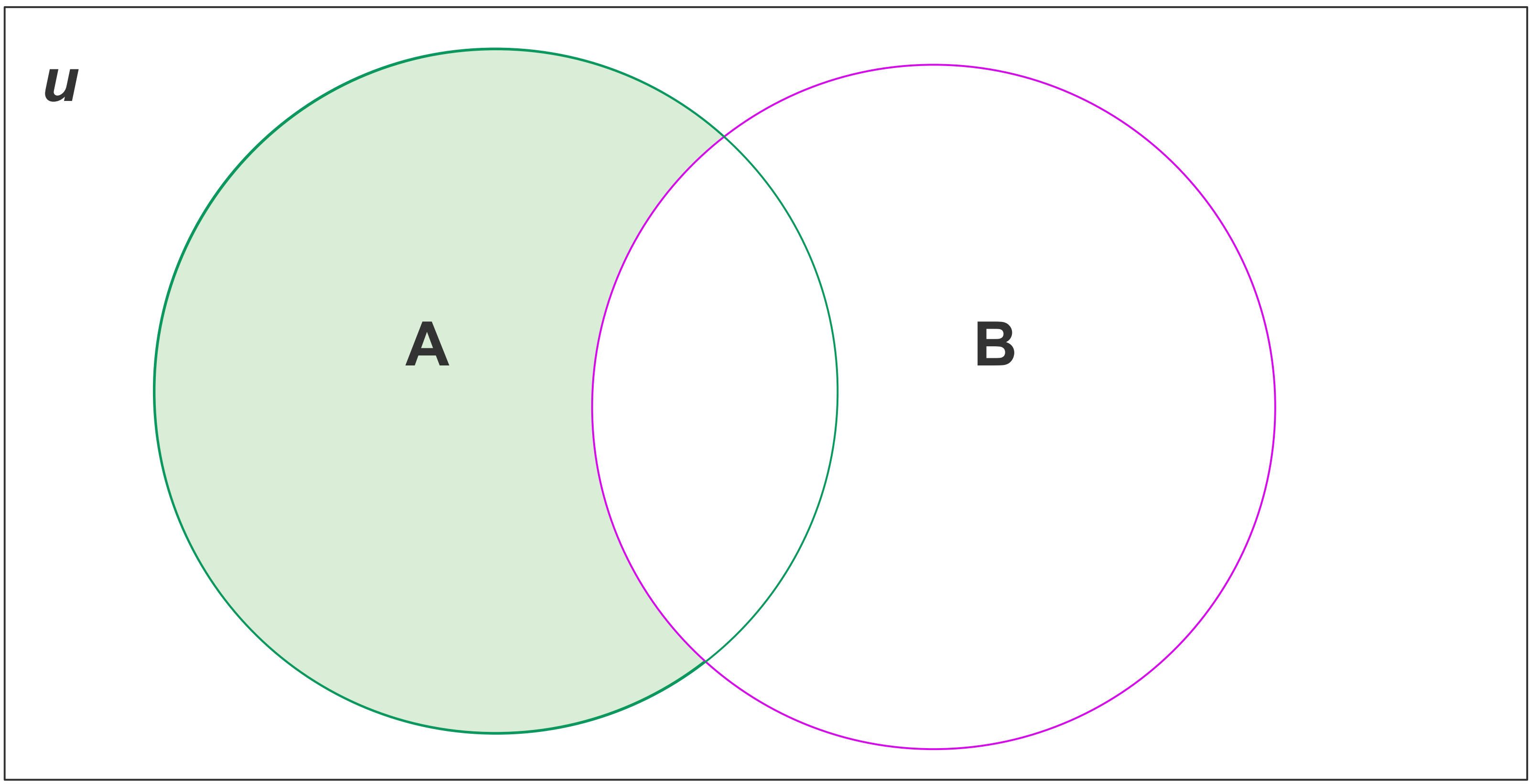
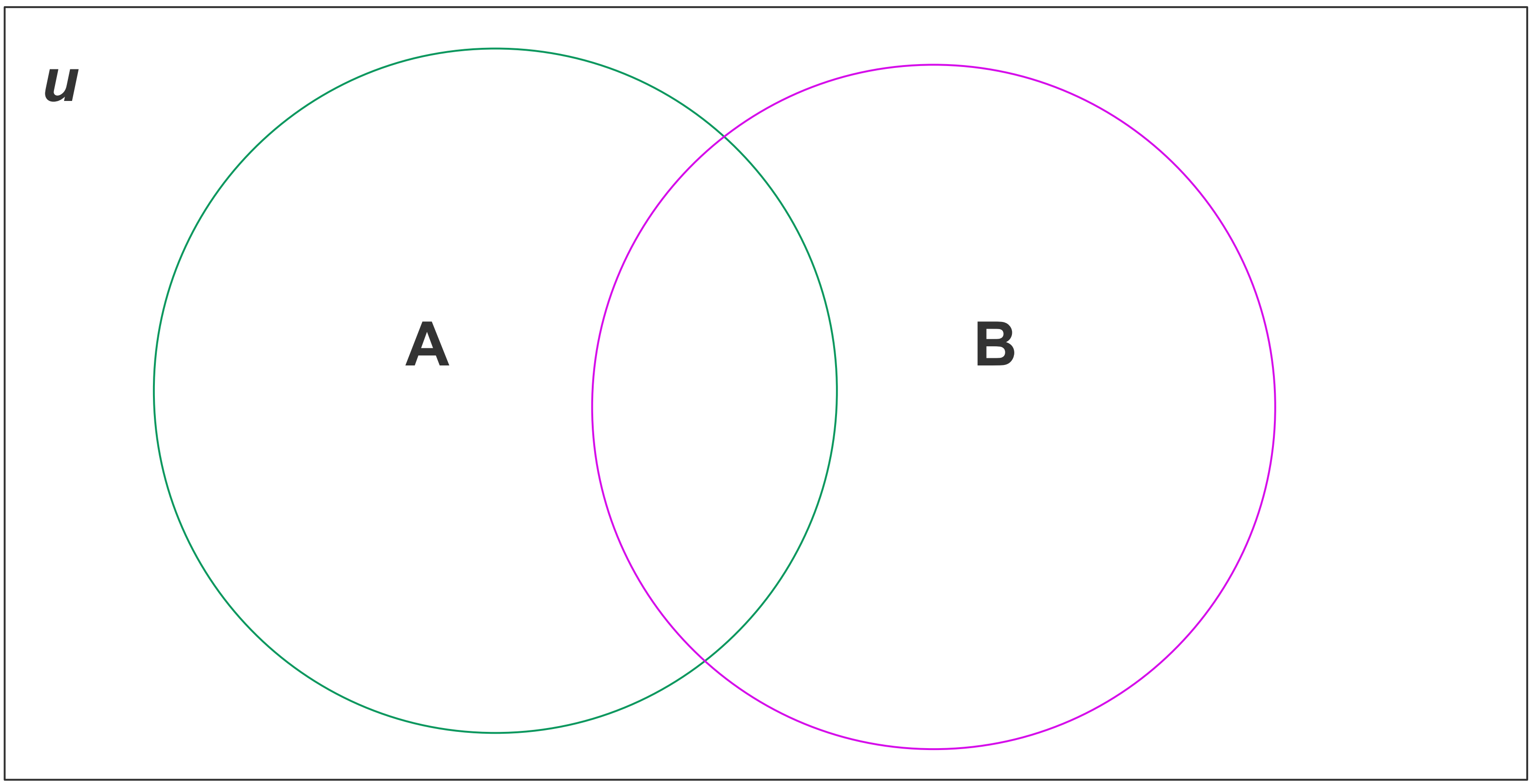


Understand Sets Visually!

What Set Operation is this?
Difference: A \ B



Understand Sets Visually!



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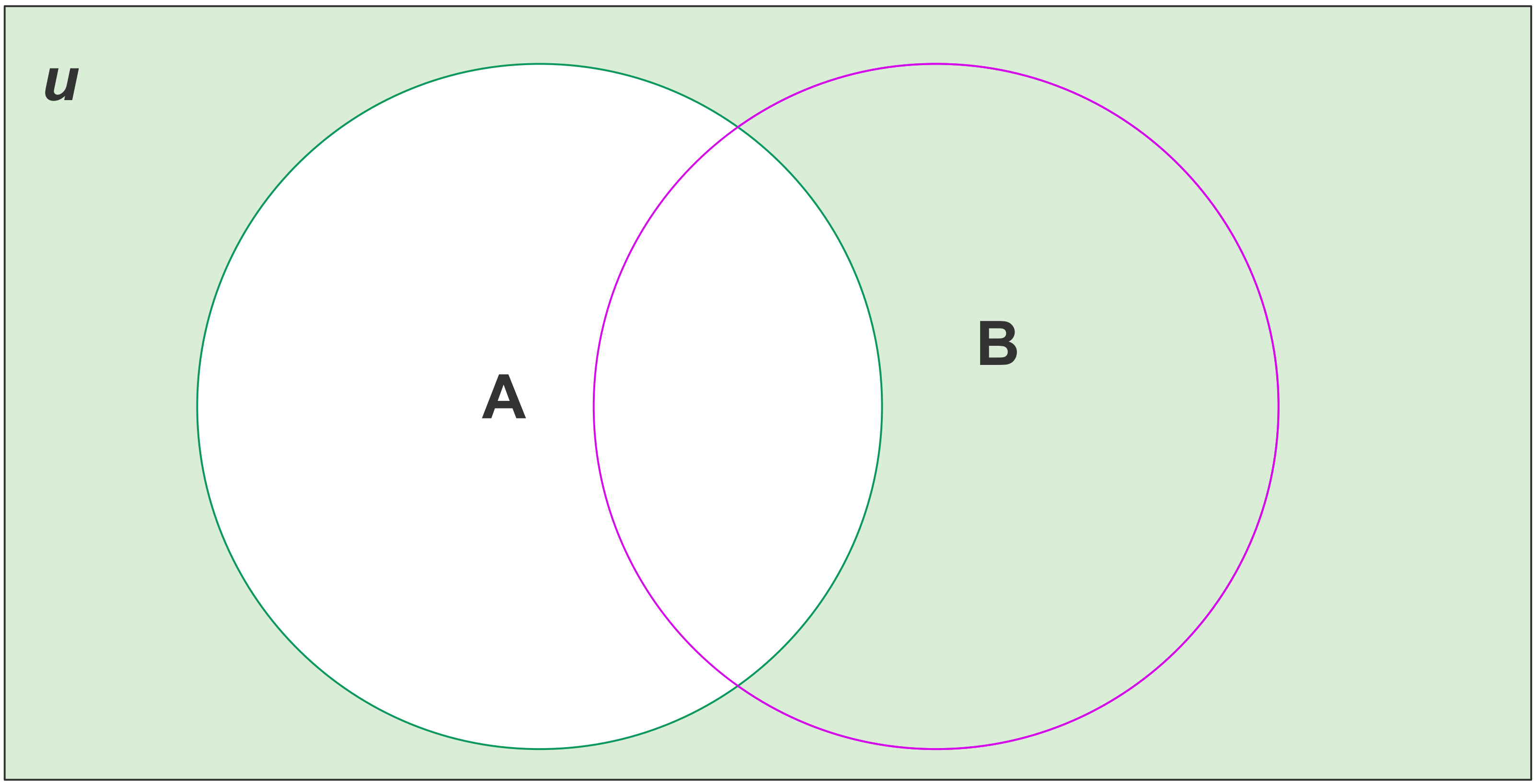
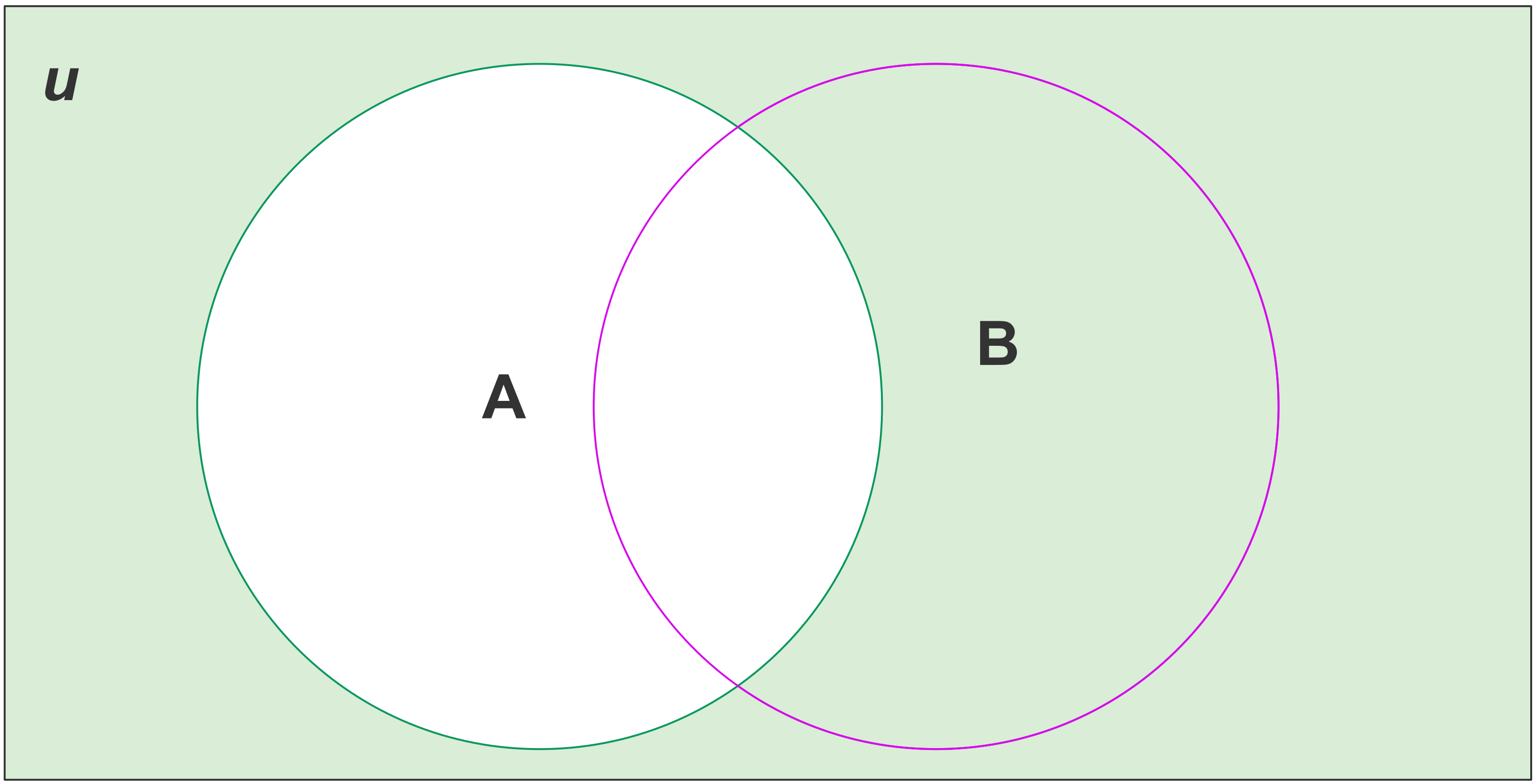


Understand Sets Visually!

What Set Operation is this?
A complement: $\overline{A}$



Cartesian Product!

Problem 2: Cartesian Product

Let A, B, C, and D be sets. Consider the following claim:

$$(A \cap B) \times C \subseteq A \times (C \cup D)$$

a) Suppose that $A = \{1, 2\}$, $B = \{1, 2, 3\}, C = \{3, 4\}, D = \{2\}$.

Calculate the values of the sets $(A \cap B) \times C$ and $A \times (C \cup D)$. Check whether the claim holds.

Cartesian Product

Let A, B, C, and D be sets. Consider the following claim:

$$(A \cap B) \times C \subseteq A \times (C \cup D)$$

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$$A \cap B = \{1, 2\}$$

Cartesian Product

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$$A \cap B = \{1, 2\}$$
$$(A \cap B) \times C = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

Cartesian Product

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Calculate the values of the sets $(A \cap B) \times C$ and $A \times (C \cup D)$. Check whether the claim holds.

$$\begin{aligned} A \cap B &= \{1, 2\} \\ (A \cap B) \times C &= \{(1, 3), (1, 4), (2, 3), (2, 4)\} \\ C \cup D &= \{2, 3, 4\} \end{aligned}$$

Cartesian Product

Let A, B, C, and D be sets. Consider the following claim:

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a) Suppose that $A = \{1, 2\}$, $B = \{1, 2, 3\}, C = \{3, 4\}, D = \{2\}$.

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$$\begin{aligned} A \cap B &= \{1, 2\} \\ (A \cap B) \times C &= \{(1, 3), (1, 4), (2, 3), (2, 4)\} \\ C \cup D &= \{2, 3, 4\} \\ A \times (C \cup D) &= \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\} \end{aligned}$$

Cartesian Product

Let A, B, C, and D be sets. Consider the following claim:

$$(A \cap B) \times C \subseteq A \times (C \cup D)$$

a) Suppose that $A = \{1, 2\}$, $B = \{1, 2, 3\}, C = \{3, 4\}, D = \{2\}$.

Calculate the values of the sets $(A \cap B) \times C$ and $A \times (C \cup D)$. Check whether the claim holds.

$$\begin{aligned} A \cap B &= \{1, 2\} \\ (A \cap B) \times C &= \{(1, 3), (1, 4), (2, 3), (2, 4)\} \\ C \cup D &= \{2, 3, 4\} \\ A \times (C \cup D) &= \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\} \end{aligned}$$

We can see that $(A \cap B) \times C \subseteq A \times (C \cup D)$. The claim holds.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

c) Write an **English proof** that the claim holds.

Follow the structure of our template for subset proofs.

Note: even though we want you to write your proof directly in English, it must still look like the translation of a formal proof. In particular, you must include all steps that would be required of a formal proof, excepting only those that we have explicitly said are okay to skip in English proofs.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

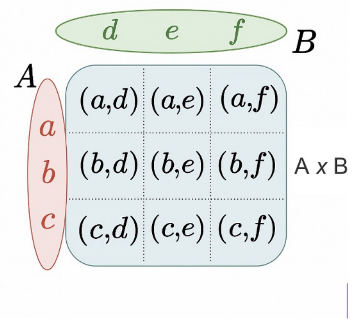
Let ***x*** be an **arbitrary** object.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

Remember, **x** is just a coordinate point!



Let ***x*** be an **arbitrary** object.

Suppose that $x \in (A \cap B) \times C$.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

Let ***x*** be an **arbitrary** object.

Suppose that $x \in (A \cap B) \times C$

Then, by definition of Cartesian product, there is some $y \in A \cap B$ and $c \in C$ such that $x = (y, c)$.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

Let ***x*** be an **arbitrary** object.

Suppose that $x \in (A \cap B) \times C$

Then, by definition of Cartesian product, there is some $y \in A \cap B$ and $c \in C$ such that $x = (y, c)$.

Then, by the definition of intersection, $y \in A$ and $y \in B$.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

Work a step back!

Let ***x*** be an **arbitrary** object.

Suppose that $x \in (A \cap B) \times C$

Then, by definition of Cartesian product, there is some $y \in A \cap B$ and $c \in C$ such that $x = (y, c)$.

Then, by the definition of intersection, $y \in A$ and $y \in B$.

Since $y \in A$ and $c \in C \cup D$, we can see that $(y, c) \in A \times (C \cup D)$ by the definition of Cartesian product.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Cartesian Product

$(A \cap B) \times C \subseteq A \times (C \cup D)$

Let ***x*** be an **arbitrary** object.

Suppose that $x \in (A \cap B) \times C$

Then, by definition of Cartesian product, there is some $y \in A \cap B$ and $c \in C$ such that $x = (y, c)$.

Then, by the definition of intersection, $y \in A$ and $y \in B$.

Also note that, by the definition of union, we can state that $c \in C \cup D$ since $c \in C$.

Since $y \in A$ and $c \in C \cup D$, we can see that $(y, c) \in A \times (C \cup D)$ by the definition of Cartesian product.

Since ***x*** was **arbitrary**, we have shown that $(A \cap B) \times C \subseteq A \times (C \cup D)$ by the definition of subset.

Structural Induction

Idea of Structural Induction

Every **element is built up recursively…**

So to show $P(s)$ for all s…

Show $P(b)$ for all base case elements b.

Show for an arbitrary element not in the base case, if $P()$ holds for every named element in the recursive rule, then $P()$ holds for the new element (each recursive rule will be a case of this proof).

Structural Induction Template

Let $P(x)$ be “<predicate>”. We show $P(x)$ holds for all x by structural induction.

Base Case: Show $P(x)$
[Do that for every base cases x.]

Inductive Hypothesis: Suppose $P(x)$ for an arbitrary x
[Do that for every x listed as in S in the recursive rules.]

Inductive Step: Show $P()$ holds for y.
[You will need a separate case/step for every recursive rule.]

Therefore $P(x)$ holds for all x by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be “”.
We show P(L) holds for all L … by structural induction on L.

<u>Base Case:</u> Show P(L) (for all L in the basis rules)

<u>Inductive Hypothesis:</u> Suppose P(L) (for all L in the recursive rules), i.e. (IH in terms of P(L))

<u>Inductive Step:</u> Goal: Show that P(?) holds. (IS goal in terms of P(?))

<u>Conclusion:</u> Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be "len(echo-pos(L)) ≤ 2 len(L)".
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(L) (for all L in the basis rules)

Inductive Hypothesis: Suppose P(L) (for all L in the recursive rules), i.e. (IH in terms of P(L))

Inductive Step: Goal: Show that P(?) holds. (IS goal in terms of P(?))

Conclusion: Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be “len(echo-pos(L)) ≤ 2 len(L)”.
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(nil)

len(echo-pos(nil)) = len(nil)

So P(nil) holds.

Inductive Hypothesis: Suppose P(L) (for all L in the recursive rules),
i.e. (IH in terms of P(L))

Inductive Step: Goal: Show that P(?) holds. (IS goal in terms of P(?))

Conclusion: Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be “len(echo-pos(L)) ≤ 2 len(L)”.
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(nil)

len(echo-pos(nil)) = len(nil)

So P(nil) holds.

Inductive Hypothesis: Suppose P(L) holds for an arbitrary L ∊ List

Inductive Step: Goal: Show that P(?) holds. (IS goal in terms of P(?))

Conclusion: Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be "len(echo-pos(L)) $\leq$ 2 len(L)".
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(nil)

len(echo-pos(nil)) = len(nil)

So P(nil) holds.

Inductive Hypothesis: Suppose P(L) holds for an arbitrary L $\in$ List

Inductive Step: Goal: Show that P(a::L) holds.

Conclusion: Therefore P(L) holds for all L $\in$ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be "len(echo-pos(L)) ≤ 2 len(L)".
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(nil)

len(echo-pos(nil)) = len(nil)

So P(nil) holds.

Inductive Hypothesis: Suppose P(L) holds for an arbitrary L ∈ List

Inductive Step: Goal: Show that P(a::L) holds. Let a ∈ Z be arbitrary.

Conclusion: Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Let P(L) be "len(echo-pos(L)) ≤ 2 len(L)".
We show P(L) holds for all lists L by structural induction on L.

Base Case: Show P(nil)

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So P(nil) holds.

Inductive Hypothesis: Suppose P(L) holds for an arbitrary L ∊ List

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.
Suppose that a ≤ 0

We can do casework!

Suppose that a > 0

Conclusion: Therefore P(L) holds for all L ∈ S by the principle of induction.

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let $a \in Z$ be arbitrary.
Suppose that $a \leq 0$

Suppose that $a > 0$

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.

Suppose that a ≤ 0

len(echo-pos(a::L)) =

Suppose that a > 0

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∈ Z be arbitrary.

Suppose that $a \le 0$

len(echo-pos(a::L)) = len(a::echo-pos(L)) Def of echo-pos (since $a \le 0$)

Suppose that $a > 0$

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.
Suppose that a ≤ 0

len(echo-pos(a::L)) = len(a::echo-pos(L)) Def of echo-pos (since a ≤ 0)

= 1 + len(echo-pos(L)) Def of len

Suppose that a > 0

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.
Suppose that a ≤ 0

len(echo-pos(a::L))	= len(a::echo-pos(L))	Def of echo-pos (since a ≤ 0)
	= 1 + len(echo-pos(L))	Def of len
	≤ 1 + 2len(L)	IH

Suppose that a > 0

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.

Suppose that a ≤ 0

len(echo-pos(a::L)) = len(a::echo-pos(L)) Def of echo-pos (since a ≤ 0)

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≤ 1 + 2len(L) IH

≤ 2 + 2len(L)

Suppose that a > 0

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.

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= 1 + len(echo-pos(L)) Def of len

≤ 1 + 2len(L) IH

≤ 2 + 2len(L)

= 2(1 + len(L))

Suppose that a > 0

Problem 5 - Structural Induction

Inductive Step: Goal: Show that P(a::L) holds. Let a ∊ Z be arbitrary.

Suppose that $a \leq 0$

len(echo-pos(a::L)) = len(a::echo-pos(L)) Def of echo-pos (since $a \leq 0$)

= 1 + len(echo-pos(L)) Def of len

≤ 1 + 2len(L) IH

≤ 2 + 2len(L)

= 2(1 + len(L))

= 2(len(a::L)) Def of len

Suppose that $a > 0$

Problem 5 - Structural Induction

<u>Inductive Step:</u> Goal: Show that $P(a::L)$ holds. Let $a \in \mathbb{Z}$ be arbitrary.

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= 1 + len(echo-pos(L)) Def of len

≤ 1 + 2len(L) IH

≤ 2 + 2len(L)

= 2(1 + len(L))

= 2(len(a::L)) Def of len

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≤ 1 + 2len(L) IH

≤ 2 + 2len(L)

= 2(1 + len(L))

= 2(len(a::L))

Def of len

Suppose that a > 0

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= 2(1 + len(L))

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Suppose that a > 0

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= 1 + len(echo-pos(L)) Def of len
≤ 1 + 2len(L) IH
≤ 2 + 2len(L)
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≤ 2 + 2len(L) IH

That’s all Folks!

By: Aruna

Extra time: Powerset English Proof

Problem 4

Let A and B be sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ follows from $A \subseteq B$.

Problem 4

Let A and B be sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ follows from $A \subseteq B$.

Let X be an arbitrary set in $\mathcal{P}(A)$.

Problem 4

Let A and B be sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ follows from $A \subseteq B$.

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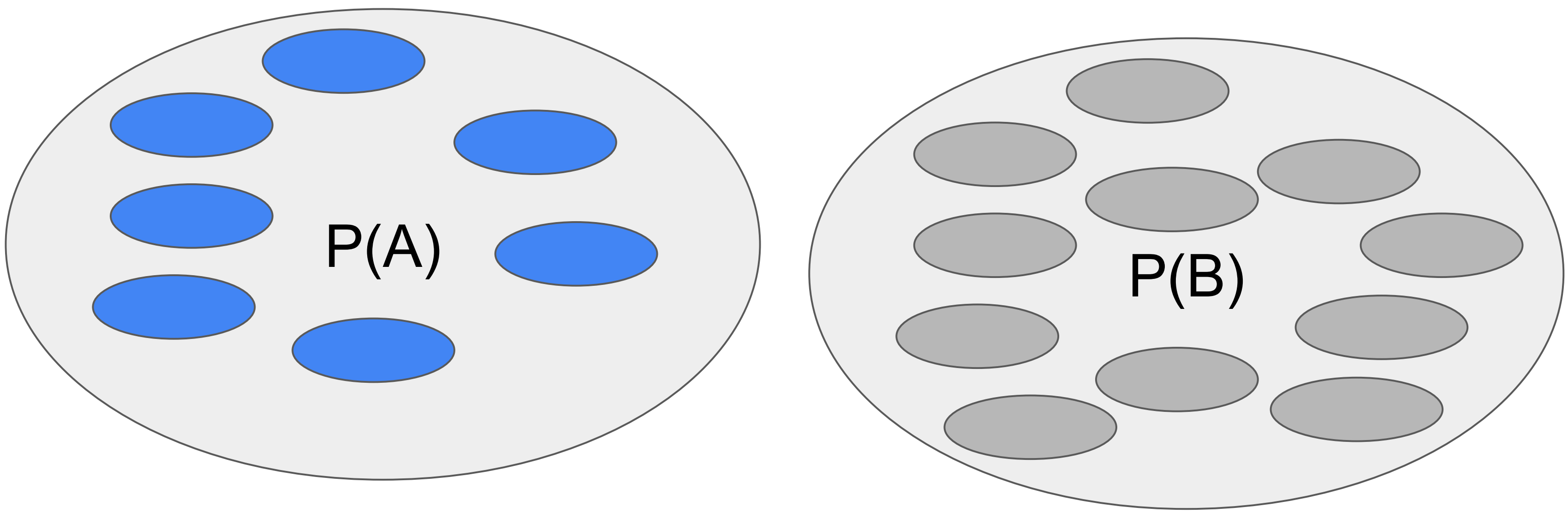
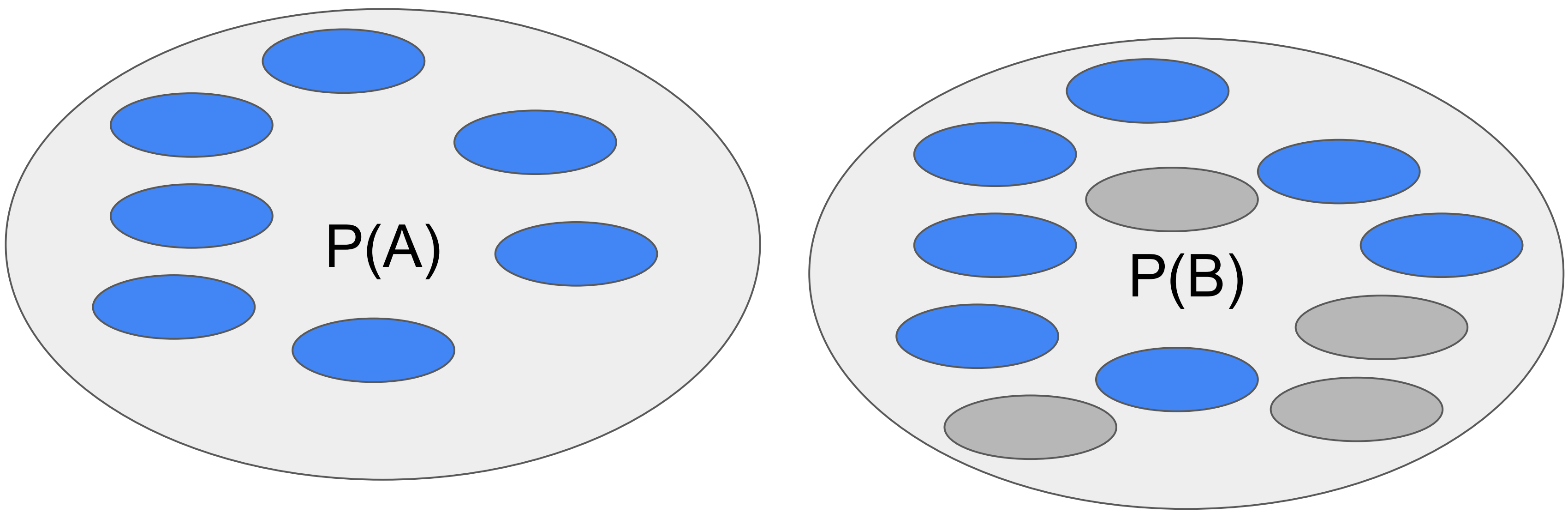
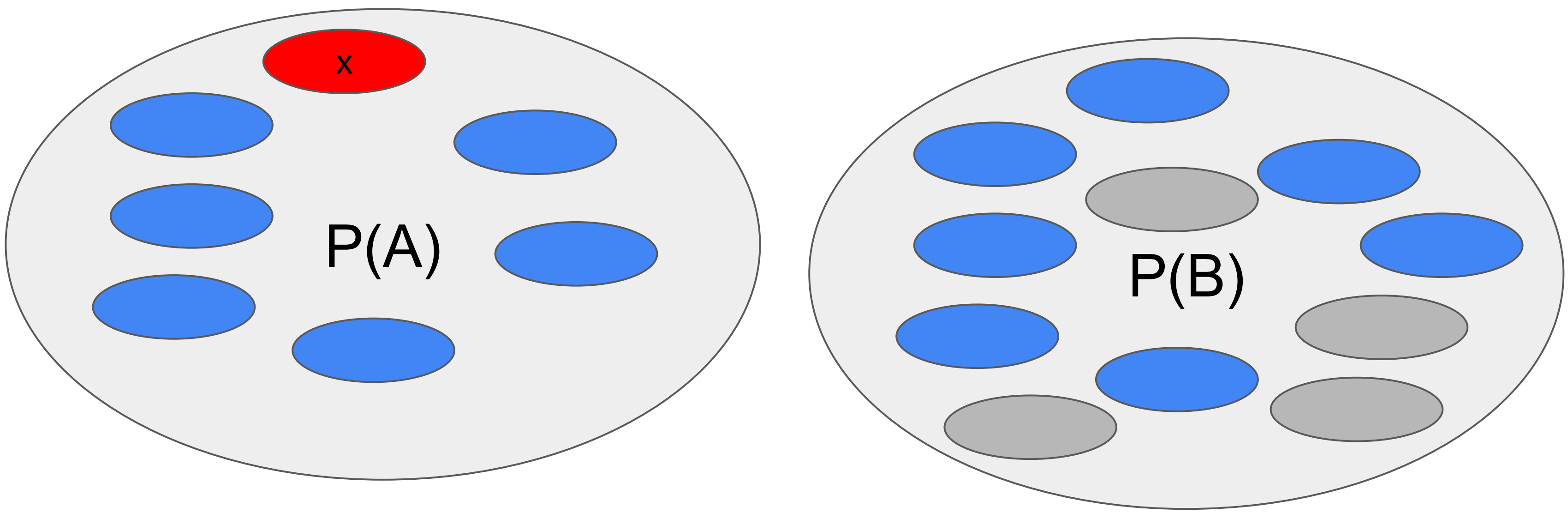
Given $A \subseteq B$, we want $P(A) \subseteq P(B)$



To show P(A) ⊆ P(B), show that the (set) elements of P(A) can be found in P(B)

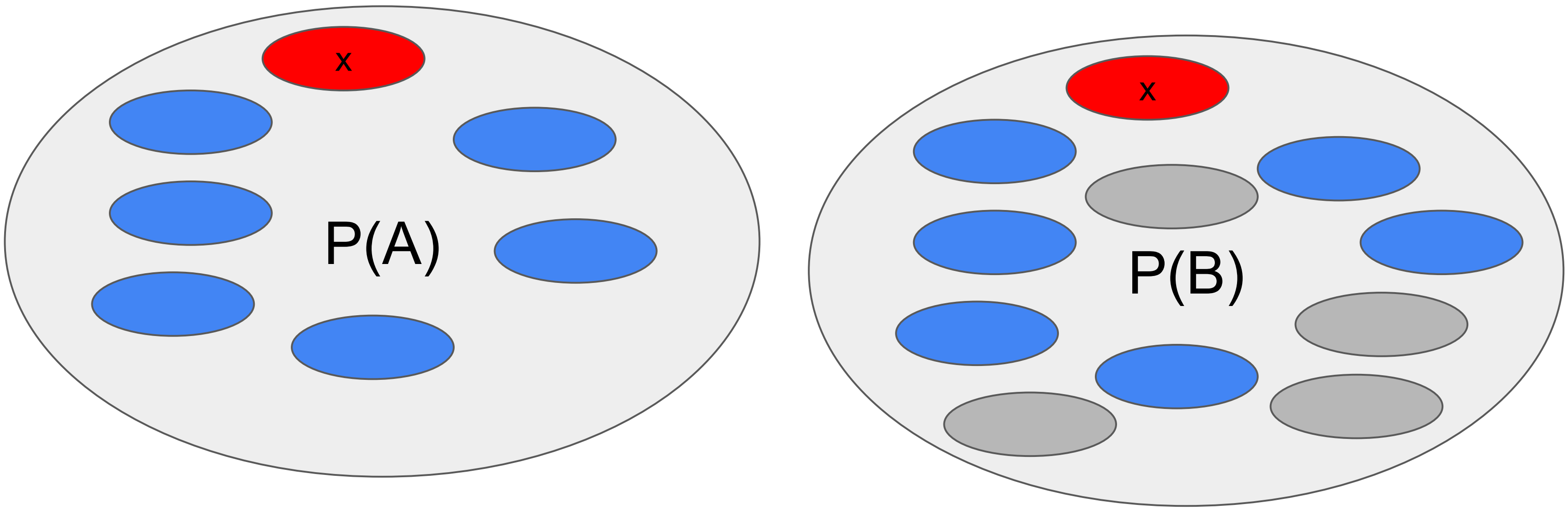


Subset proof strategy: take an arbitrary element x of P(A)...

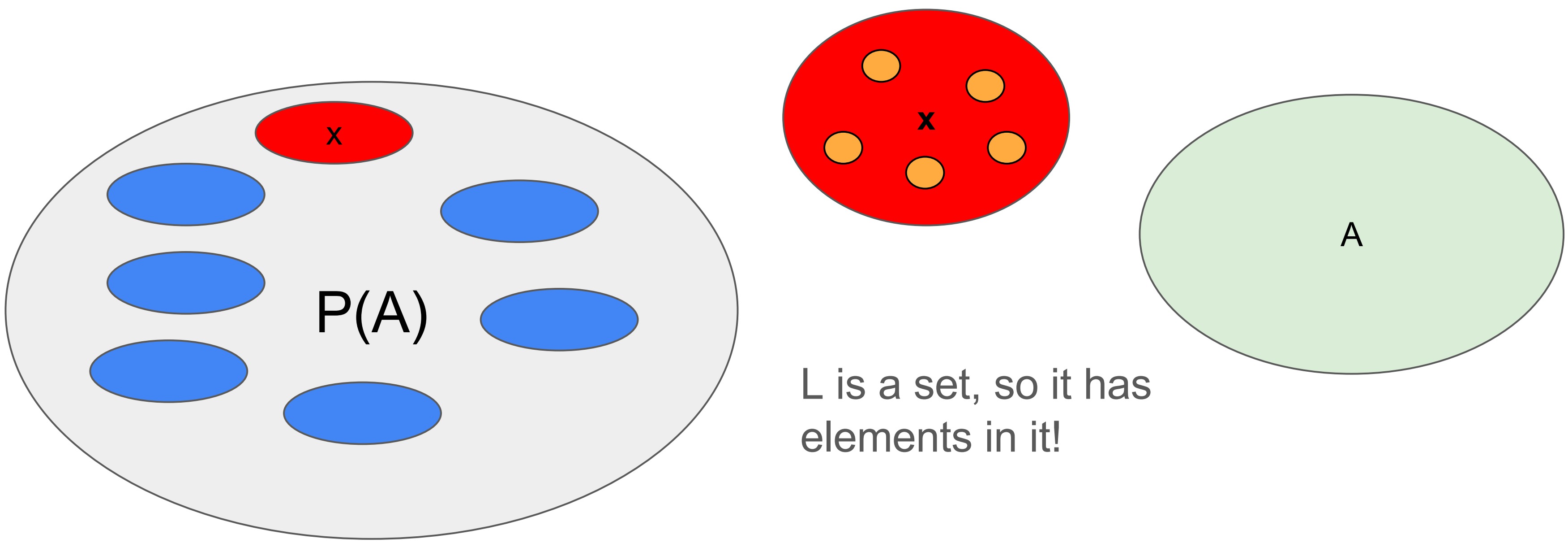


Subset proof strategy: … and show that it's in P(B)

How do we show x is in P(B)?



Well, x is in P(A), so x ⊆ A by definition of powerset. Our target is showing x is in P(B), i.e., x ⊆ B.



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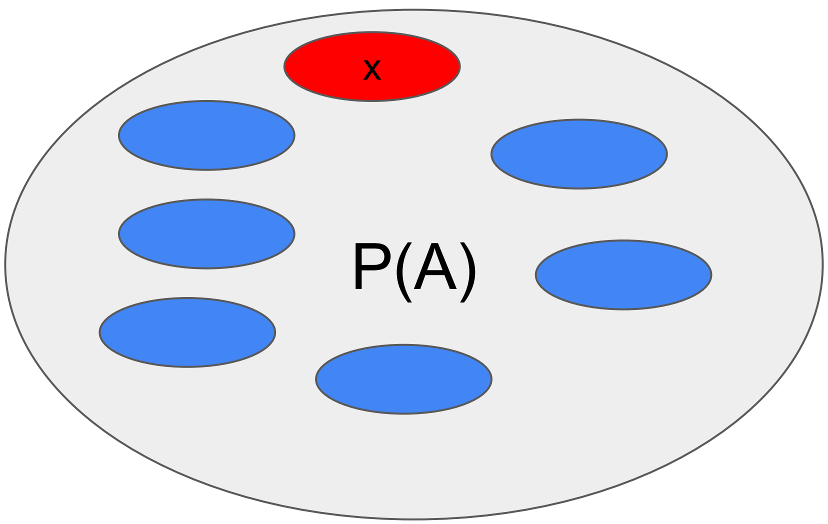
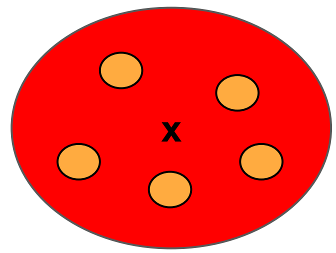
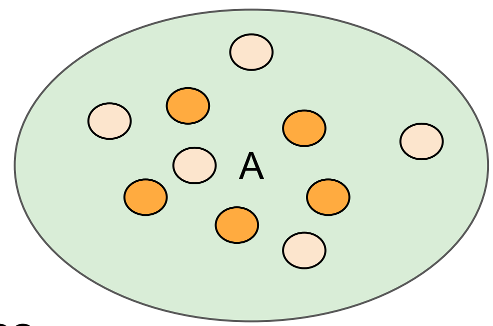
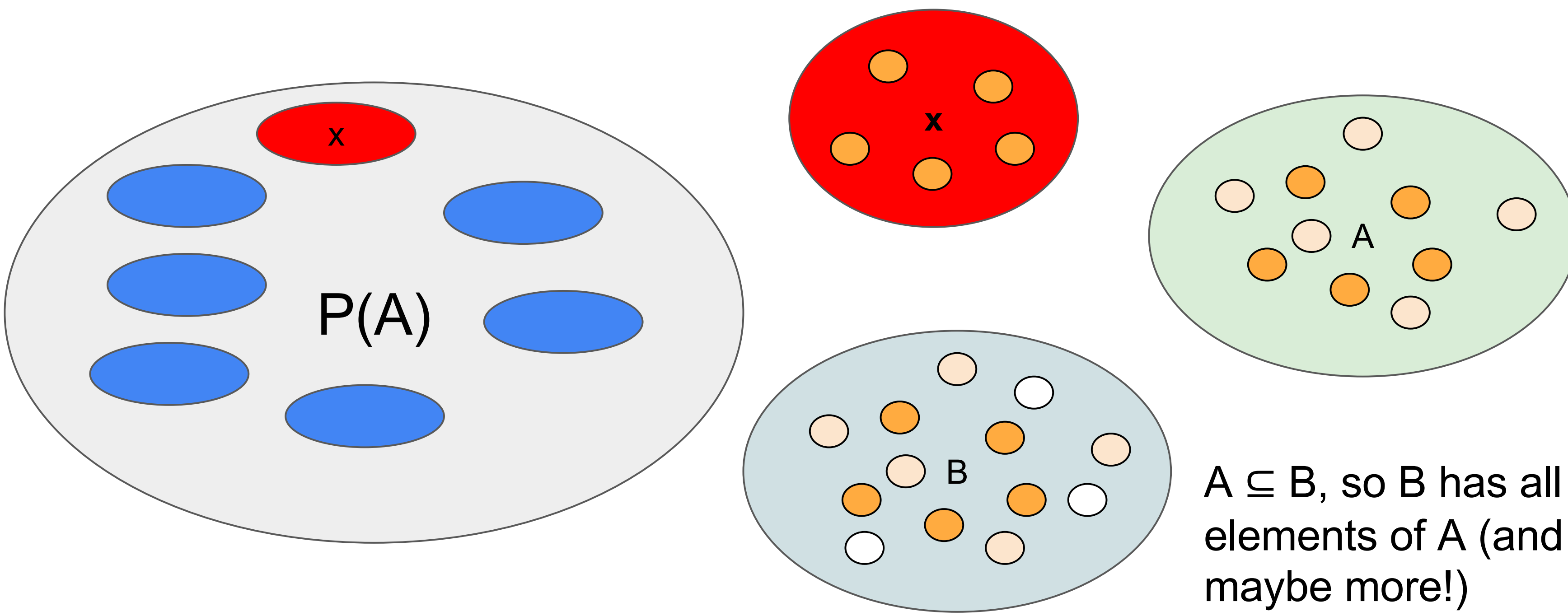






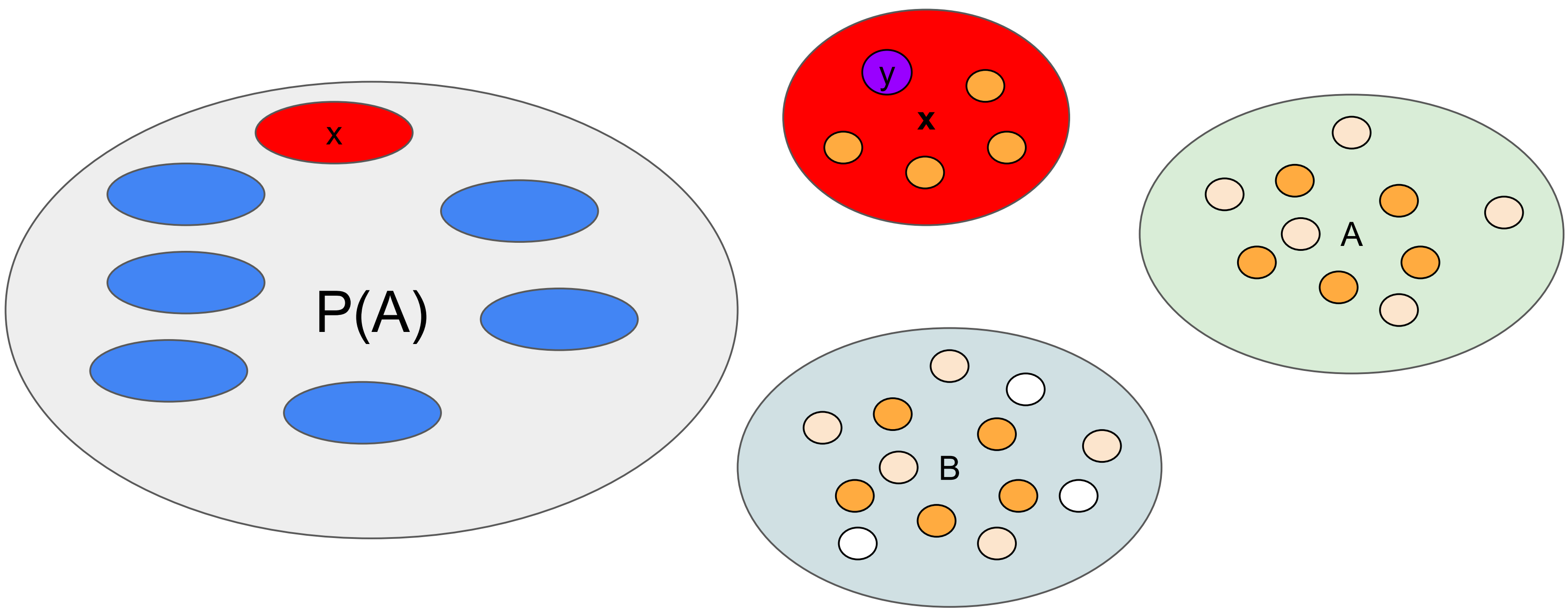
Since L ⊆ A, A has those elements too (and maybe more stuff!)

Well, x is in P(A), so x ⊆ A. Our target is showing x is in P(B), i.e., x ⊆ B.

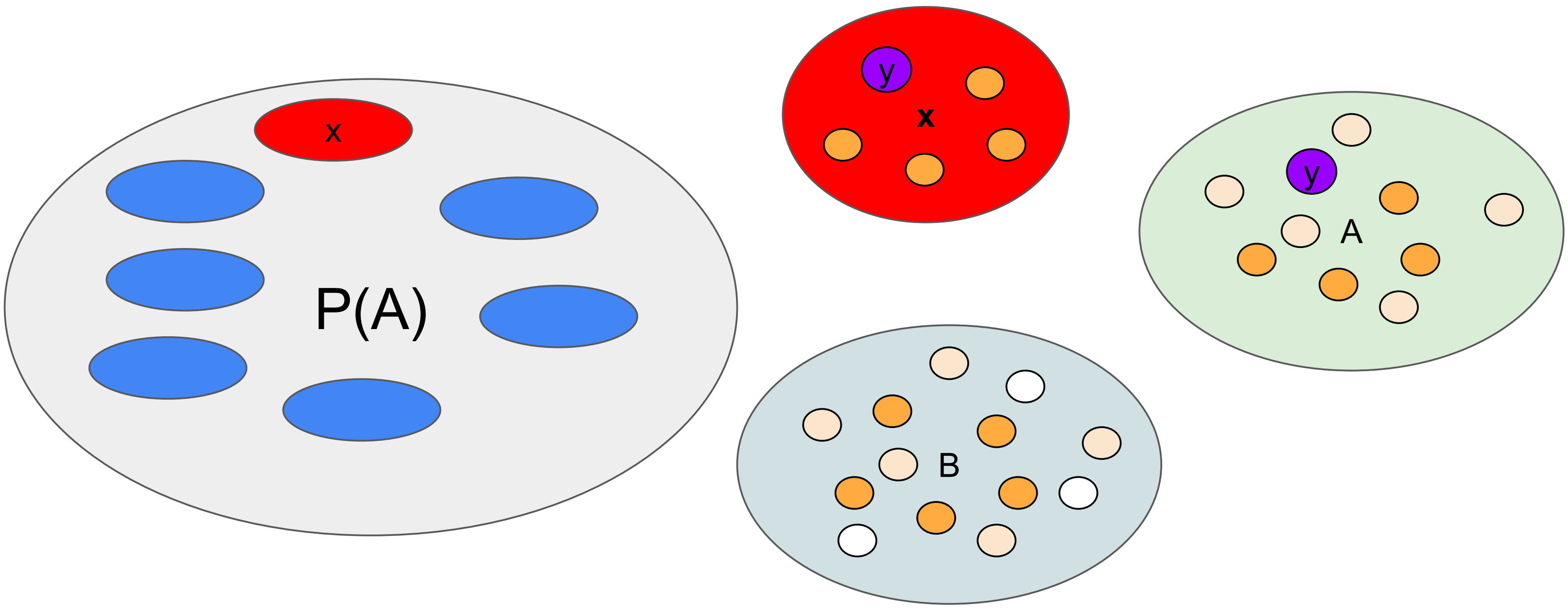


A ⊆ B, so B has all the elements of A (and maybe more!)

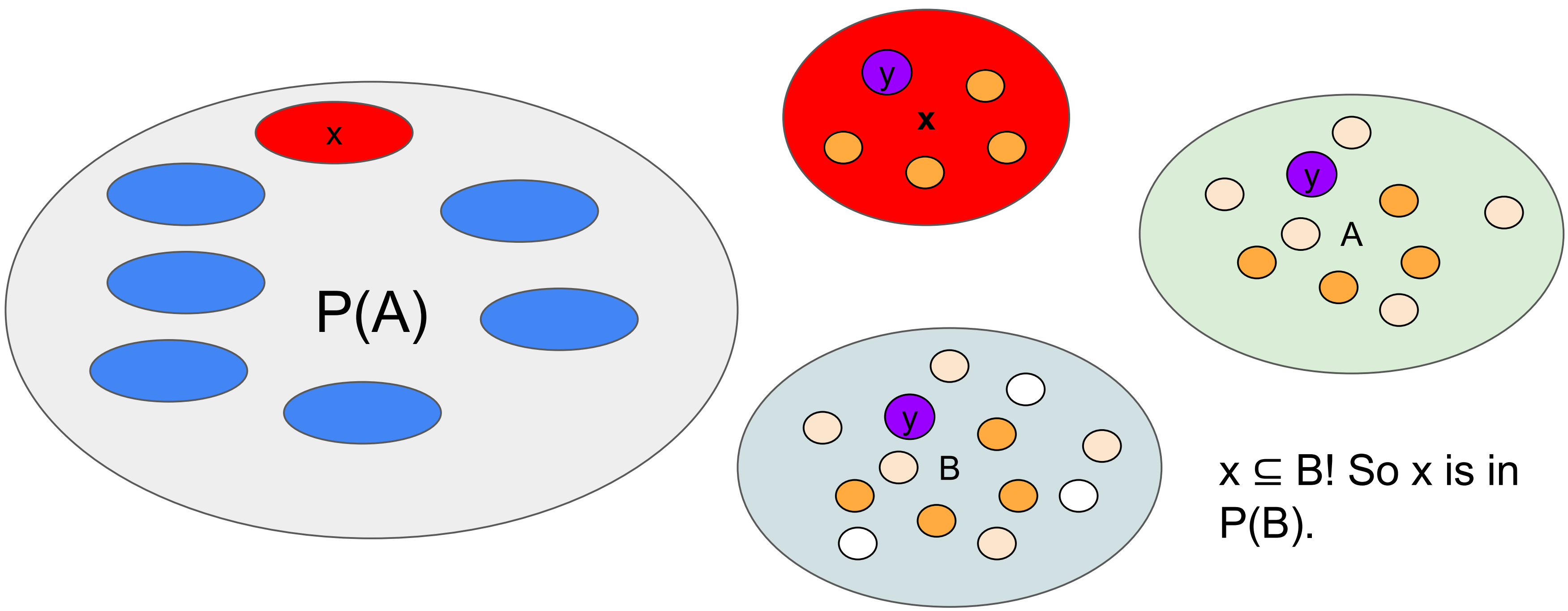
To show x ⊆ B, we do the subset strategy again: take an arbitrary y in x…



To show x ⊆ B, we do the subset strategy again: Since x ⊆ A, y is in A…



To show x ⊆ B, we do the subset strategy again: And finally since A ⊆ B, y is in B.



x ⊆ B! So x is in P(B).

Cozy Set Proofs

Using Cozy For Sets

- **A U B**: A Union B- “A cup B”
- **A ∩ B**: “A cap B”
- **A ∊ B**: “A in B”
- A \ B: “A \ B”
- B complement- “~B” (Only one Argument)
- A\B\C is implicitly (A\B)\C

Problem 2a – Subsets

For any sets A, B, and C, show that it holds that $A \setminus B \subseteq A \cup C$

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Let x be arbitrary.

1.1.1.	$x \in A \setminus B$	assumption
1.1.2.	$x \in A \land \neg(x \in B)$	Def of Set Difference 1.1.1

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For any sets A, B, and C, show that it holds that $A \setminus B \subseteq A \cup C$

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1.1.1.	$x \in A \setminus B$	assumption
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1.1.3.	$x \in A$	Elim And 1.1.2

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1.1.4.	$x \in A \lor x \in B$	Intro or 1.1.3

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1.1.4.	$x \in A \vee x \in B$	Intro or 1.1.3
1.1.5.	$x \in A \cup B$	Def of Union 1.1.4

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1.1.	$x \in A \setminus B \rightarrow x \in A \cup B$	Direct Proof

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1.1.	$x \in A \setminus B \rightarrow x \in A \cup B$	Direct Proof
1.	$\forall x, x \in A \setminus B \rightarrow x \in A \cup B$	Intro forall

Problem 2a – Subsets

For any sets A, B, and C, show that it holds that $A \backslash B \subseteq A \cup C$

Let x be arbitrary.

1.1.1.	$x \in A \backslash B$	assumption
1.1.2.	$x \in A \wedge \neg(x \in B)$	Def of Set Difference 1.1.1
1.1.3.	$x \in A$	Elim And 1.1.2
1.1.4.	$x \in A \vee x \in B$	Intro or 1.1.3
1.1.5.	$x \in A \cup B$	Def of Union 1.1.4
1.1.	$x \in A \backslash B \rightarrow x \in A \cup B$	Direct Proof
1.	$\forall x, x \in A \backslash B \rightarrow x \in A \cup B$	Intro forall
2.	$A \backslash B \subseteq A \cup B$	Def of Subset 1

Problem 2a – Subsets

For any sets A, B, and C, show that it holds that $A \setminus B \subseteq A \cup C$

Let x be arbitrary.

1.1.1.	$x \in A \setminus B$	assumption
1.1.2.	$x \in A \wedge \neg(x \in B)$	Def of Set Difference 1.1.1
1.1.3.	$x \in A$	Elim And 1.1.2
1.1.4.	$x \in A \vee x \in B$	Intro or 1.1.3
1.1.5.	$x \in A \cup B$	Def of Union 1.1.4
1.1.	$x \in A \setminus B \rightarrow x \in A \cup B$	Direct Proof
1.	$\forall x, x \in A \setminus B \rightarrow x \in A \cup B$	Intro forall
2.	$A \setminus B \subseteq A \cup B$	Def of Subset 1

Let x be an arbitrary object. Suppose that $x \in A \setminus B$. By definition, this means that $x \in A$ and $x \notin B$. Since $x \in A$, we have $x \in A \cup C$ by the definition of $\cup$. Since x was arbitrary, this shows $A \setminus B \subseteq A \cup C$.

Problem 2a – Cozy (posted with solutions)

For any sets A, B, and C, show that it holds that $A \setminus B \subseteq A \cup C$

Let x be arbitrary.

1.1.1.	`x in A \ B`	`assumption`
1.1.2.	`x in A and not (x in B)`	`defof \ {A} {B} 1.1.1`
1.1.3.	`x in A`	`elim and 1.1.2 left`
1.1.4.	`x in A or x in B`	`intro or 1.1.3 (x in B) right`
1.1.5.	`x in A cup B`	`undef cup {A} {B} 1.1.4` ✖
1.1.	`x in A \ B -> x in A cup B`	`direct proof (x in A \ B -> x in A cup B)` ✖
1.	`forall x, x in A \ B -> x in A cup B`	`intro forall (forall x, x in A \ B -> x in A cup B) x`
2.	`A \ B subset A cup B`	`undef subset {A \ B} {A cup B} 1` ✖