# Quiz Section 6: Midterm Review

## Task 1 – Midterm Review: Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- $\texttt{soy}(x)$ is true iff $x$ contains soy milk.
- $\texttt{whole}(x)$ is true iff $x$ contains whole milk.
- $\texttt{sugar}(x)$ is true iff $x$ contains sugar
- $\texttt{decaf}(x)$ is true iff $x$ is not caffeinated.
- $\texttt{vegan}(x)$ is true iff $x$ is vegan.
- $\texttt{KevinLikes}(x)$ is true iff Kevin likes the drink $x$.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like $=$ and $\neq$.

**a)** Coffee drinks with whole milk are not vegan.

**b)** Kevin only likes one coffee drink, and that drink is not vegan.

**c)** There is a drink that has both sugar and soy milk.

Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\texttt{decaf}(x) \land \texttt{KevinLikes}(x)] \rightarrow \texttt{sugar}(x))$$

## Task 2 – Remains to be Seen

Prove the following for all integers $x, y \in \mathbb{Z}$:

$$\text{If } x \equiv_6 1 \text{ and } y \equiv_5 3 \text{ then } 5x + 3y \equiv_{15} 14.$$

**a)** Let your domain be integers. Write the predicate logic of this claim.

**b)** Write a formal proof for this claim.

## Task 3 – Formal Proof

Show that for any integers $x, y, z$, where $x + y$ is even and $y + z$ is odd that $x - z$ is odd.

**a)** Let your domain be integers. Write the predicate logic of this claim. Define predicates Odd and Even!

**b)** Write a formal proof for this claim.

## Task 4 – Bernoulli's Inequality

Prove that for $x \geqslant 0$ and even integer $n \geqslant 0$,

$$(1 + x)^n \geqslant 1 + nx.$$

Note that this is Bernoulli's inequality with slightly adjusted bounds for simplicity
**W**e will use the definition of even to write $n = 2j$ and then induct on $j$.

## Task 5 – Strong Induction

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.