CSE 311 Section MR

Midterm Review

Administrivia

Announcements & Reminders

- HW5
  - Was due yesterday @ 11:00 PM
  - Use late days if you need to!
  - Make sure you tagged pages on gradescope correctly

- Midterm is Coming Next Week!!!
  - Next Wednesday 5/14 in class (50 minutes)
  - Bring your Husky ID :)

Problem 1: Translation (review)

Problem 1 – Translation

Let your domain of discourse be **all coffee drinks.** You should use the following predicates:

- $\texttt{decaf}(x)$ is true iff x is not caffeinated.
- $\texttt{vegan}(x)$ is true iff x is vegan.
- $\texttt{KevinLikes}(x)$ is true iff Kevin likes the drink x.
- $\texttt{soy}(x)$ is true iff x contains soy milk.
- $\texttt{whole}(x)$ is true iff x contains whole milk.
- $\texttt{sugar}(x)$ is true iff x contains sugar

Work on **part (B)** with the people around you.

b) Kevin only likes one coffee drink, and that drink is not vegan

Problem 1 – Translation

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b) Kevin only likes one coffee drink, and that drink is not vegan

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes:

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃ x (KevinLikes(x))**

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃ x (KevinLikes(x))**
Only one coffee drink that Kevin likes:

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃x (KevinLikes(x))**
Only one coffee drink that Kevin likes: ∃x (KevinLikes(x)) ∧ **∀y [KevinLikes(y) → x = y**

Problem 1 – Translation

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- KevinLikes(x) is true iff Kevin likes the drink x.
- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar

b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃ x (KevinLikes(x))**
Only one coffee drink that Kevin likes: ∃ x (KevinLikes(x)) ∧ **∀ y [KevinLikes(y) → x = y**

Only one **non-vegan** coffee drink that Kevin likes:

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃x (KevinLikes(x))**
Only one coffee drink that Kevin likes: ∃x (KevinLikes(x)) ∧ **∀y [KevinLikes(y) → x = y**

Only one **non-vegan** coffee drink that Kevin likes:
∃x (KevinLikes(x) ∧ ¬Vegan(x) ∧ ∀y [KevinLikes(y) → x = y])

Problem 1 – Translation

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b) Kevin only likes one coffee drink, and that drink is not vegan

Some coffee drink that Kevin Likes: **∃x (KevinLikes(x))**
Only one coffee drink that Kevin likes: ∃x (KevinLikes(x)) ∧ **∀y [KevinLikes(y) → x = y**

Only one **non-vegan** coffee drink that Kevin likes:
∃x (KevinLikes(x) ∧ ¬Vegan(x) ∧ ∀y [KevinLikes(y) → x = y])

Alternative: ∃x ∀y (KevinLikes(x) ∧ ¬Vegan(x) ∧ [KevinLikes(y) → x = y])

Problem 2: Formal Proof (review)

Problem 2 – Remains to be Seen

Prove the following for all integers $x, y \in \mathbb{Z}$:

$$\text{If } x \equiv_6 1 \text{ and } y \equiv_5 3 \text{ then } 5x + 3y \equiv_{15} 14.$$

a) Let your domain be integers. Write the predicate logic of this claim.

Remains to be Seen

Prove the following for all integers $x, y \in \mathbb{Z}$:

If $x \equiv_6 1$ and $y \equiv_5 3$ then $5x + 3y \equiv_{15} 14$.

a) Let your domain be integers. Write the predicate logic of this claim.

$$\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Remains to be Seen

$$\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

2. $\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$ Intro $\forall$

Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \land (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

2.1.1	$(x \equiv_6 1) \land (y \equiv_5 3)$	Assumption
2.1	$[(x \equiv_6 1) \land (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
2.	$\forall x, \forall y[(x \equiv_6 1) \land (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Intro $\forall$

Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

2.1.1	$(x \equiv_6 1) \wedge (y \equiv_5 3)$	Assumption
2.1.2	$(x \equiv_6 1)$	Elim $\wedge$: 2.1.1

2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
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Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \land (y \equiv_5 3)] \to 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

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2.1.3	$(y \equiv_5 3)$	Elim $\land$: 2.1.1
2.1.4	$6 \mid 1 - x$	Def of Congruent: 2.1.2

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2.	$\forall x, \forall y[(x \equiv_6 1) \land (y \equiv_5 3)] \to (5x + 3y \equiv_{15} 14)$	Intro $\forall$

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2.1.5	$5 \mid 3 - y$	Def of Congruent: 2.1.3
2.1.6	$\exists k, 6k = (1 - x)$	Def of Divides: 2.1.4

2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
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2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6

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2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6
2.1.9	$5b = (3 - y)$	Elim $\exists$: 2.1.7

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2.1.6	$\exists k, 6k = (1 - x)$	Def of Divides: 2.1.4
2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6
2.1.9	$5b = (3 - y)$	Elim $\exists$: 2.1.7

2.1.13	$5x + 3y \equiv_{15} 14$	Undef Congruent: 2.1.12
2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
2.	$\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Intro $\forall$

Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

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2.1.5	$5 \mid 3 - y$	Def of Congruent: 2.1.3
2.1.6	$\exists k, 6k = (1 - x)$	Def of Divides: 2.1.4
2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6
2.1.9	$5b = (3 - y)$	Elim $\exists$: 2.1.7

2.1.12	$15 \mid 14 - 5x - 3y$	Undef Divides: 2.1.11
2.1.13	$5x + 3y \equiv_{15} 14$	Undef Congruent: 2.1.12
2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
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Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

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2.1.6	$\exists k, 6k = (1 - x)$	Def of Divides: 2.1.4
2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6
2.1.9	$5b = (3 - y)$	Elim $\exists$: 2.1.7
2.1.11	$\exists k, 15k = 14 - 5x - 3y$	Intro $\exists$: 2.1.10
2.1.12	$15 \mid 14 - 5x - 3y$	Undef Divides: 2.1.11
2.1.13	$5x + 3y \equiv_{15} 14$	Undef Congruent: 2.1.12
2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
2.	$\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Intro $\forall$

Remains to be Seen

$$\forall x \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow 5x + 3y \equiv_{15} 14$$

Let x and y be arbitrary integers.

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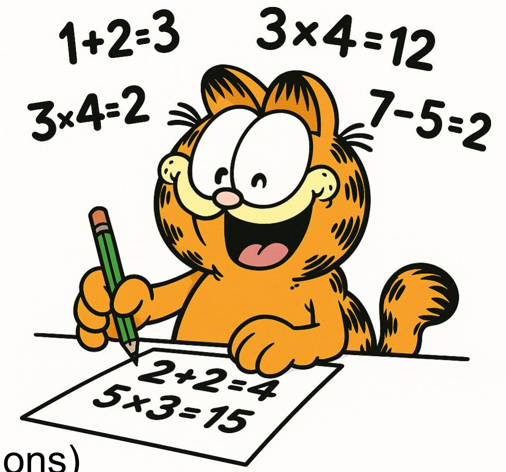
2.1.7	$\exists k, 5k = (3 - y)$	Def of Divides: 2.1.5
2.1.8	$6a = (1 - x)$	Elim $\exists$: 2.1.6
2.1.9	$5b = (3 - y)$	Elim $\exists$: 2.1.7
2.1.10	$15(2a + b) = 14 - 5x - 3y$	Algebra
2.1.11	$\exists k, 15k = 14 - 5x - 3y$	Intro $\exists$: 2.1.10
2.1.12	$15 \mid 14 - 5x - 3y$	Undef Divides: 2.1.11
2.1.13	$5x + 3y \equiv_{15} 14$	Undef Congruent: 2.1.12
2.1	$[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Direct Proof
2.	$\forall x, \forall y[(x \equiv_6 1) \wedge (y \equiv_5 3)] \rightarrow (5x + 3y \equiv_{15} 14)$	Intro $\forall$

Problem 4: Induction (review)

Quick Tip: Inductive Steps

🛡️ **Play it S.A.F.E.R in Induction Steps**



S — Simple Algebra

Remember the basic truths.

Example: $3 = 2 + 1$ or $0 = 1 - 1$

A — Apply known facts (like the Inductive Hypothesis or recursive definitions)

F — Factor out powers or expressions (sometimes this is not obvious)

Example: $3x(x + 1 + 4x^2) + 2\,(x + 1 + 4x^2) = (3x+2)(x + 1 + 4x^2)$

E — Extract constants or patterns

Exponent example: $2^{k+1} = 2 \cdot 2^k$

Summation example: $$\sum_{i=1}^{k+1} i = \left(\sum_{i=1}^{k} i\right) + (k+1)$$

R— Reverse

Stuck? Work a step back from the end goal!

Bernoulli's Inequality

Prove that for **$x \geq 0$** and even integer **$n \geq 0$***

$$(1 + x)^n \geq 1 + nx$$

Hint: Use the definition of even to write $n = 2j$ and then induct on j

*note that this is the relaxed case of the actual bounds Bernoulli's Inequality is defined for

Bernoulli's Inequality

$(1 + x)^n \geq 1 + nx.$

Since *n* is even, we can write $n = 2j$ for some integer $j \geq 0$. **We will prove this by induction on *j*.**
Define ***P(j)* := "$(1 + x)^{2j} \geq 1 + 2jx$"** and prove using induction that *P(j)* holds for all integers $j \geq 0$, $x \geq 0$

Bernoulli's Inequality

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Base Case. If $j = 0$

Bernoulli's Inequality

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Define ***P(j)* := "$(1 + x)^{2j} \geq 1 + 2jx$"** and prove using induction that $P(j)$ holds for all integers $j \geq 0$, $x \geq 0$

Base Case. If $j = 0$ then,

$$(1 + x)^{2j} = (1 + x)^0 = 1$$

Bernoulli's Inequality

$(1 + x)^n \geq 1 + nx.$

Since n is even, we can write $n = 2j$ for some integer $j \geq 0$. **We will prove this by induction on *j*.**
Define ***P(j)*** **:= "$(1 + x)^{2j} \geq 1 + 2jx$"** and prove using induction that $P(j)$ holds for all integers $j \geq 0$, $x \geq 0$

Base Case. If $j = 0$ then,

$$(1 + x)^{2j} = (1 + x)^0 = 1 \geq 1 + (0)x = 1 + 2jx.$$

Bernoulli's Inequality

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Base Case. If $j = 0$ then,

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So the claim holds for $j = 0$.

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Base Case. If $j = 0$ then,

$$(1 + x)^{2j} = (1 + x)^0 = 1 \geq 1 + (0)x = 1 + 2jx.$$

So the claim holds for $j = 0$.

Inductive Hypothesis. Suppose that $P(k)$ holds for some arbitrary nonnegative integer k. [***P(k)*** **:= "$(1 + x)^{2k} \geq 1 + 2kx$"**]

Bernoulli's Inequality

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So the claim holds for $j = 0$.

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$\geq (1 + 2kx)(1 + x)^2$ Inductive Hypothesis

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$$= (1 + 2kx)(1 + 2x + x^2)$$
$$\geq (1 + 2kx)(1 + 2x) \qquad \text{since } x^2 \geq 0$$

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$$
\begin{aligned}
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\end{aligned}$$

Which proves $P(k + 1)$, note we used the inductive hypothesis, the fact $P(k)$ is true on the second line.
Thus $P(j)$ holds for all non-negative integers j by the principle of induction. Therefore, we also have that $P(n)$ holds.

That’s All! Good luck studying <3

Problem 5: Strong Induction (extra time)

Task 5: Strong Induction

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.

Strong Induction

Let ***P(n)* be "*a(n)* = 2*n* − 1".** We will show that *P(n)* is true for all $n \geq 1$ by strong induction.

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Base Cases ($n = 1$, $n = 2$):

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$a(1) = 1 = 2 \cdot 1 - 1$

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So, *P(1)* and *P(2)* hold.

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Inductive Step:

We will show $P(k + 1)$ holds.

$a(k + 1) =$

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$a(k + 1) = 2a(k) - a(k - 1)$ [Definition of *a*]

$= 2(k + 1) - 1$

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$a(k + 1) = 2a(k) - a(k - 1)$ [Definition of a]

$= 2(2k - 1) - (2(k - 1) - 1)$ [Inductive Hypothesis]

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So, $P(k + 1)$ holds.

Conclusion:

Therefore, $P(n)$ holds for all integers $n \geq 1$ by the principle of strong induction.