Quiz Section 3: Inference

Review

Inference Rules

$$\frac{}{\therefore A \vee \neg A} \quad \text{(Excluded Middle)}$$

$$\frac{A; B}{\therefore A \wedge B} \quad \text{(Intro } \wedge\text{)} \qquad \frac{A \wedge B}{\therefore A, B} \quad \text{(Elim } \wedge\text{)}$$

$$\frac{A, B}{\therefore A \vee B} \quad \text{(Intro } \vee\text{)} \qquad \frac{A \vee B; \neg A}{\therefore B} \quad \text{(Elim } \vee\text{)}$$

$$\frac{A \Rightarrow B}{\therefore A \to B} \quad \text{(Direct Proof)} \qquad \frac{A; A \to B}{\therefore B} \quad \text{(Modus Ponens)}$$

$$\frac{P(c) \text{ for some } c}{\therefore \exists x\, P(x)} \quad \text{(Intro } \exists\text{)} \qquad \frac{\forall x\, P(x)}{\therefore P(a) \text{ for any } a} \quad \text{(Elim } \forall\text{)}$$

$$\frac{\text{Let } a \text{ be arbitrary} \ldots P(a)}{\therefore \forall x\, P(x)} \quad \text{(Intro } \forall\text{)}$$

(If no other name in P depends on a)

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some special } c} \quad \text{(Elim } \exists\text{)} \quad \text{list dependencies for } c$$

Task 1 – Formal Proof I

a) Given $(q \wedge r), (r \to \neg s)$ and $(s \vee p)$, show that p holds.

b) Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

Task 2 – Formal Proof II

Show that $\neg p$ follows from $\neg(\neg r \vee k)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$ with a formal proof. You can try this problem on Cozy at `https://bit.ly/cse311-formalproof2`.

Task 3 – Direct Proof

a) Show that $\neg k \to s$ follows from $k \vee q$, $q \to r$ and $r \to s$ with a formal proof. You can try this problem on Cozy at `https://bit.ly/cse311-directproof`

b) Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

Task 4 – Proof, Goof, or Spoof I

For each of the claims below, (1) translate the English proof into a formal proof and (2) say which of the following categories describes the formal proof:

Proof The proof is correct.

Goof The claim is true but the proof is wrong.

Spoof The claim is false.

Finally, (3) if it is a goof, point out the errors in the proof and explain how to correct them, and if it is a spoof, point out the *first* error in the proof and then show that the claim is false by giving a counterexample. (If it is a correct proof, then skip part (3).)

a) Show that r follows from $\neg p$ and $p \leftrightarrow r$.

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

b) Show that $p \rightarrow q$ follows from $r \vee \neg p$ and $r \rightarrow q$.

Proof, Goof, or Spoof: Assume p is true. Since we are given that $r \vee \neg p$, this is the same as $\neg p \vee r$, which means we know $p \rightarrow r$. We assumed that p holds, which means we now know r holds. We are also given that $r \rightarrow q$, which means q holds. Now since if we assumed if p holds, q must hold, we know that $p \rightarrow q$ must hold.

Task 5 – Predicate Logic Formal Proof

a) Given $\forall x\,(T(x) \to M(x))$, we wish to prove $(\exists x\, T(x)) \to (\exists y\, M(y))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\,(T(x) \to M(x))$ ____________________
    - 2.1. $\exists x\, T(x)$ ____________________
    - 2.2. $T(c)$ ____________________
    - 2.3. $T(c) \to M(c)$ ____________________
    - 2.4. $M(c)$ ____________________
    - 2.5. $\exists y\, M(y)$ ____________________
2. $(\exists x\, T(x)) \to (\exists y\, M(y))$ ____________________

b) Given $\forall x\, \exists y\,(T(x) \to S(y,x))$, we wish to prove $\exists x\,(T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\exists x\,(T(x) \to \forall y\, S(y,x))$ ____________________
2. $T(c) \to \forall y\, S(y,c)$ ____________________
    - Let a be arbitrary.
        - 3.1.1. $T(c)$ ____________________
        - 3.1.2. $\forall y\, S(y,c)$ ____________________
        - 3.1.3. $S(a,c)$ ____________________
    - 3.1. $T(c) \to S(a,c)$ ____________________
    - 3.2. $\exists x\,(T(x) \to S(a,x))$ ____________________
3. $\forall y\, \exists x\,(T(x) \to S(y,x))$ ____________________

Task 6 – Proof, Goof, or Spoof?

For each of the claims below, (1) translate the English proof into a formal proof and (2) say which of the following categories describes the formal proof:

Proof The proof is correct.

Goof The claim is true but the proof is wrong.

Spoof The claim is false.

Finally, (3) if it is a goof, point out the errors in the proof and explain how to correct them, and if it is a spoof, point out the *first* error in the proof and then show that the claim is false by giving a counterexample. (If it is a correct proof, then skip part (3).)

a) Show that $\exists z\ \forall x\ P(x, z)$ follows from $\forall x\ \exists y\ P(x, y)$.

Proof, Goof, or Spoof: We are given that, for every x, there is some y such that P(x, y) holds. Thus, there must be some object c such that for every x, P(x, c) holds. This shows that there exists an object z such that, for every x, P(x, z) holds.

b) Show that $\exists z\ (P(z) \wedge Q(z))$ follows from $\forall x\ P(x)$ and $\exists y\ Q(y)$.

Proof, Goof, or Spoof: Let z be arbitrary. Since we were given that for every x, P(x) holds, P(z) must hold. Since we were given that there is a y such that Q(y) holds, Q(z) must also hold. From the previous facts, we know that there is some object z such that P(z) and Q(z) hold.

c) **Claim**: Given $\forall x\, \forall y\, (R(x, y) \to \neg R(y, x))$, it follows that $\neg \exists x\, R(x, x)$.

Proof or Spoof: For contradiction, assume that $R(c, c)$ holds for some c. From the given, taking c for x and y, we get that $\neg R(c, c)$ holds. Since we cannot have both $R(c, c)$ and $\neg R(c, c)$ true, we have a contradiction. Therefore, the claim is true.