CSE 311 Section 3

Quantifiers and Proofs

Administrivia & Introductions

Announcements & Reminders

- HW1 out
  - If you think something was graded incorrectly, submit a regrade request!
  - Regrades generally will be open for a week

- HW2 was due yesterday 4/17 on Gradescope
  - Use a late day if you need to!
  - Gradescope: Make sure you select the pages for each question correctly
  - !! Selecting the pages after the deadline won't mark it as late

- HW3
  - Due Wednesday 4/24 @ 11:00pm

Formal Proofs!

Rules to Remember

Elim ∧
$$\frac{A \wedge B}{\therefore A, B}$$

Intro ∧
$$\frac{A \,;\, B}{\therefore A \wedge B}$$

Elim ∨
$$\frac{A \vee B \,;\, \neg A}{\therefore B}$$

Intro ∨
$$\frac{A}{\therefore A \vee B, B \vee A}$$

Modus Ponens
$$\frac{A \,;\, A \rightarrow B}{\therefore B}$$

Direct Proof
$$\frac{A \Rightarrow B}{\therefore A \rightarrow B}$$

Tautology
$$\frac{A \equiv T}{\therefore A}$$

Equivalent
$$\frac{A \equiv B \,;\, B}{\therefore A}$$

Rules to Remember

$$\boxed{\text{Elim } \vee} \; \frac{A \vee B \; ; \; \neg A}{\therefore B}$$

$$\boxed{\text{Cases}} \; \frac{A \vee B \; ; \; A \rightarrow C \; ; \; B \rightarrow C}{\therefore C}$$

Problem 1b - Formal Proof

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Lets get setup:

Problem 1b - Formal Proof

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Lets get setup:

1.	$a \rightarrow b$	Given
2.	$c \rightarrow b$	Given
3.	$a \vee (c \wedge d)$	Given
	b	

Problem 1b - Formal Proof

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Initial observation:

1.	$a \rightarrow \boxed{b}$	Given
2.	$c \rightarrow \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given

$\boxed{b}$

Problem 1b - Formal Proof

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

Initial observation:
if we get a or c,
then can get to b

1.	$\boxed{a} \rightarrow \boxed{b}$	Given
2.	$\boxed{c} \rightarrow \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given
	$\boxed{b}$	

Problem 1b - Formal Proof

Given $(a \rightarrow b)$, $(c \rightarrow b)$, $a \vee (c \wedge d)$, show that b holds.

We can work a step back!

1.	$\boxed{a} \rightarrow \boxed{b}$	Given
2.	$\boxed{c} \rightarrow \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given

$\boxed{(a \vee c)}$

$\boxed{b}$

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

What is this called?

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given

$$\boxed{(a \vee c)}$$

$$\boxed{b}$$

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

What is this called?

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given

$\boxed{(a \vee c)}$

$\boxed{b}$

$\boxed{\text{Cases: } \times\ 1, 2}$

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

How can we get

$\boxed{(a \vee c)}$

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given
	$\boxed{(a \vee c)}$	
	$\boxed{b}$	Cases: X 1, 2

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

Distributivity!

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$\boxed{(a \vee c)} \wedge (a \vee d)$	Distributivity: 3
5.	$\boxed{(a \vee c)}$	
6.	$\boxed{b}$	$\boxed{\text{Cases: X 1, 2}}$

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

What's missing?

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$\boxed{(a \vee c)} \wedge (a \vee d)$	Distributivity: 3
5.	$\boxed{(a \vee c)}$	
6.	$\boxed{b}$	Cases: X 1, 2

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \lor (c \land d)$, show that b holds.

We did it!

1.	$\boxed{a} \to \boxed{b}$	Given
2.	$\boxed{c} \to \boxed{b}$	Given
3.	$a \lor (c \land d)$	Given
4.	$\boxed{(a \lor c)} \land (a \lor d)$	Distributivity: 3
5.	$\boxed{(a \lor c)}$	Elim $\land$: 4
6.	$\boxed{b}$	$\boxed{\text{Cases: } 5, 1, 2}$

Problem 1b - Formal Proof

Given $(a \to b)$, $(c \to b)$, $a \vee (c \wedge d)$, show that b holds.

1.	$a \to b$	Given
2.	$c \to b$	Given
3.	$a \vee (c \wedge d)$	Given
4.	$(a \vee c) \wedge (a \vee d)$	Distributivity: 3
5.	$(a \vee c)$	Elim $\wedge$: 4
6.	b	Cases: 5, 1, 2

Direct Proofs!

Direct Proof

$$\frac{A \Rightarrow B}{\therefore A \to B}$$

Introduce an assumption like a new variable when you are conducting an experiment…

You will typically need this new assumption because your Givens alone are not sufficient

Problem 3b

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

Just the setup:

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]

$r \to p$

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude if $\boxed{r}$ then $\boxed{p}$?

$\boxed{r}$ does not exist alone…

(2) contains r but we cannot elim or here…

$$\boxed{r} \to \boxed{p}$$

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude if $\boxed{r}$ then $\boxed{p}$?

$\boxed{r}$ does not exist alone…

Could we assume $\boxed{r}$?

$$\boxed{r} \to \boxed{p}$$

Problem 3

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]

How do we conclude
if r then p?

r does not exist alone…

Could we assume r?
Yes! Let's use **direct proof rule**!

$$\boxed{r} \rightarrow \boxed{p}$$

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4	$\boxed{p}$	
	$\boxed{r} \to \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

Since we have $\boxed{r}$, can we use line 2?

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \to (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4	$\boxed{p}$	
	$\boxed{r} \to \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

Since we have r, can we use line 2?
Almost! Let's create the left hand side of line 2

1.	$p \vee \neg q$	[Given]
2.	$\boxed{(r \vee s)} \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4.2.	$\boxed{r \vee s}$	[$\vee$intro, 4.1]
4	$\boxed{p}$	
	$\boxed{r} \rightarrow \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

Next: Modus Ponens!

1.	$p \vee \neg q$	[Given]
2.	$\boxed{(r \vee s)} \rightarrow (q \vee s)$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4.2.	$\boxed{r \vee s}$	[$\vee$intro, 4.1]
4	$\boxed{p}$	
	$\boxed{r} \rightarrow \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

Next: Modus Ponens!

1.	$p \vee \neg q$		[Given]
2.	$\boxed{(r \vee s)} \to \boxed{(q \vee s)}$		[Given]
3.	$\neg s$		[Given]
4.1.	$\boxed{r}$	[Assumption]	
4.2.	$\boxed{r \vee s}$	[$\vee$intro, 4.1]	
4.3.	$\boxed{q \vee s}$	[MP 4.2, 2]	
4	$\boxed{p}$		
	$\boxed{r} \to \boxed{p}$		[Direct Proof Rule]

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

We should use **q** to get to $\boxed{p}$…
How can we get **q** alone?

1.	$p \vee \neg q$		[Given]
2.	$(r \vee s) \to \boxed{(q \vee s)}$		[Given]
3.	$\neg s$		[Given]
4.1.	$\boxed{r}$	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4.3.	$\boxed{q \vee s}$	[MP 4.2, 2]	
4	$\boxed{p}$	[$\vee$elim, 4.5, 1]	
	$\boxed{r} \to \boxed{p}$		[Direct Proof Rule]

Problem 3

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

We should use **q** to get to $\boxed{p}$…

use elim or!

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow \boxed{(q \vee s)}$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4.2.	$r \vee s$	[$\vee$intro, 4.1]
4.3.	$\boxed{q \vee s}$	[MP 4.2, 2]
4.4.	$\boxed{q}$	[$\vee$elim, 4.3, 3]
4	$\boxed{p}$	
	$\boxed{r} \rightarrow \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \rightarrow p$ follows from $p \vee \neg q$, $(r \vee s) \rightarrow (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

We should use **q** to get to $\boxed{p}$…

use double negation!

1.	$p \vee \neg q$	[Given]
2.	$(r \vee s) \rightarrow \boxed{(q \vee s)}$	[Given]
3.	$\neg s$	[Given]
4.1.	$\boxed{r}$	[Assumption]
4.2.	$r \vee s$	[$\vee$intro, 4.1]
4.3.	$\boxed{q \vee s}$	[MP 4.2, 2]
4.4.	$\boxed{q}$	[$\vee$elim, 4.3, 3]
4.5.	$\neg\neg q$	[equivalent, 4.4]
4	$\boxed{p}$	
	$\boxed{r} \rightarrow \boxed{p}$	[Direct Proof Rule]

Problem 3

Show that $r \to p$ follows from $p \vee \neg q$, $(r \vee s) \to (q \vee s)$, and $\neg s$.

How do we conclude $\boxed{p}$?

We should use **q** to get to $\boxed{p}$...

now we can use line 1!

1.	$\boxed{p \vee \neg q}$		[Given]
2.	$(r \vee s) \to \boxed{(q \vee s)}$		[Given]
3.	$\neg s$		[Given]
4.1.	$\boxed{r}$	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4.3.	$\boxed{q \vee s}$	[MP 4.2, 2]	
4.4.	$\boxed{q}$	[$\vee$elim, 4.3, 3]	
4.5.	$\neg\neg q$	[equivalent, 4.4]	
4.6.	$\boxed{p}$	[$\vee$elim, 4.5, 1]	
	$\boxed{r} \to \boxed{p}$		[Direct Proof Rule]

Problem 3

1.	$p \vee \neg q$		[Given]
2.	$(r \vee s) \rightarrow (q \vee s)$		[Given]
3.	$\neg s$		[Given]
4.1.	r	[Assumption]	
4.2.	$r \vee s$	[$\vee$intro, 4.1]	
4.3.	$q \vee s$	[MP 4.2, 2]	
4.4.	q	[$\vee$elim, 4.3, 3]	
4.5.	$\neg\neg q$	[equivalent, 4.4]	
4.6.	p	[$\vee$elim, 4.5, 1]	
4.	$r \rightarrow p$		[Direct Proof Rule]

Notes on Cozy:

- Video Tutorials are Linked on Course Website
- Important: If you leave Cozy open for hours on one problem, saving errors will occur (cse cookies only last for a few hours)

Spoof/Proof/Goof

Problem 4a

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

1. $p \leftrightarrow r$ Given

Just the setup

r

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

1.	$p \leftrightarrow r$	Given
	$p \rightarrow r$	
	r	

Can we jump to p → q ?

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

1.	$p \leftrightarrow r$	Given
2.	$(p \to r) \wedge (r \to p)$	Defn Biconditional: 1
	$p \to r$	Elim $\wedge$: 2
	r	

Can we jump to p → q ?

No! We need to use the **Definition of Biconditional + Elim AND**

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
	$\neg p$	Given
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

What is the reasoning behind this?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
	$\neg p \vee (p \vee r)$	
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

Intro OR

1.	$p \leftrightarrow r$	Given
2.	$(p \to r) \wedge (r \to p)$	Defn Biconditional: 1
3.	$p \to r$	Elim $\wedge$: 2
4.	$\neg p$	Given
	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to (p ∧ ¬p) → r?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
	$(p \wedge \neg p) \rightarrow r$	
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to (p ∧ ¬p) → r?

No! we need a few steps:

1.	$p \leftrightarrow r$	Given
2.	$(p \to r) \wedge (r \to p)$	Defn Biconditional: 1
3.	$p \to r$	Elim $\wedge$: 2
4.	$\neg p$	Given
	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
	$(p \wedge \neg p) \to r$	
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to $(p \wedge \neg p) \rightarrow r$?

No! we need a few steps:

Associativity

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5

$(p \wedge \neg p) \rightarrow r$

r

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to (p ∧ ¬p) → r?

No! we need a few steps:

Working a step back now…

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
	$\neg(p \wedge \neg p) \vee r$	
	$(p \wedge \neg p) \rightarrow r$	Law of Implication:
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to (p ∧ ¬p) → r?

No! we need a few steps:

Looks Like Demorgan's!
We need Double negation first…

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
	$\neg(p \wedge \neg p) \vee r$	
	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 9
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Can we jump to (p ∧ ¬p) → r?

No! we need a few steps:

Now DeMorgan's!

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8

r

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

What rule is this using?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8
	$\neg p \rightarrow r$	
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \to r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \lor (p \lor r)$ holds. This claim is equivalent to $(p \land \neg p) \to r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \to r$. Since we were given that $\neg p$ holds, we get that r holds.

What rule is this using?

Elim AND?

1.	$p \leftrightarrow r$	Given
2.	$(p \to r) \land (r \to p)$	Defn Biconditional: 1
3.	$p \to r$	Elim $\land$: 2
4.	$\neg p$	Given
5.	$\neg p \lor (p \lor r)$	Intro $\lor$: 4
6.	$(\neg p \lor p) \lor r$	Associativity: 5
7.	$(\neg p \lor \neg\neg p) \lor r$	Double Negation: 6
8.	$\neg(p \land \neg p) \lor r$	DeMorgans: 7
9.	$(p \land \neg p) \to r$	Law of Implication: 8
	$\neg p \to r$	Elim $\land$: 9
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

What rule is this using?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8
	$\neg p \rightarrow r$	Elim $\wedge$: 9
	r	

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8
	$\neg p \rightarrow r$	Elim $\wedge$: 9
11.	r	Modus Ponens: 4, 10

What rule is this using?

Modus Ponens!

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

But is this proof correct?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8
10.	$\neg p \rightarrow r$	Elim $\wedge$: 9
11.	r	Modus Ponens: 4, 10

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Elim AND is used on a subexpression which is <u>incorrect</u>

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$\underline{(p \wedge \neg p)} \rightarrow r$	Law of Implication: 8
10.	$\neg p \rightarrow r$	$\boxed{\text{Elim } \wedge\text{: 9}}$
11.	r	Modus Ponens: 4, 10

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Is the conclusion correct?

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$\underline{(p \wedge \neg p)} \rightarrow r$	Law of Implication: 8
10.	$\neg p \rightarrow r$	Elim $\wedge$: 9
11.	r	Modus Ponens: 4, 10

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

Is the conclusion correct?

No! You cannot conclude **r**

you can only conclude **¬r** (using ¬p)

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$(p \wedge \neg p) \rightarrow r$	Law of Implication: 8
10.	$\neg p \rightarrow r$	$\boxed{\text{Elim } \wedge\text{: 9}}$
11.	r	Modus Ponens: 4, 10

Problem 4

Proof, Goof, or Spoof: Since we are given that $p \leftrightarrow r$, we know $p \rightarrow r$. We are also given that $\neg p$ holds, so it must be the case that $\neg p \vee (p \vee r)$ holds. This claim is equivalent to $(p \wedge \neg p) \rightarrow r$. Since this last claim starts by assuming both p and $\neg p$, we can infer that this holds with just $\neg p$, giving us $\neg p \rightarrow r$. Since we were given that $\neg p$ holds, we get that r holds.

This is Spoof!

Counterexample:

p := **false**
r := **false**
¬p := **true**
p ↔ r := **true**

1.	$p \leftrightarrow r$	Given
2.	$(p \rightarrow r) \wedge (r \rightarrow p)$	Defn Biconditional: 1
3.	$p \rightarrow r$	Elim $\wedge$: 2
4.	$\neg p$	Given
5.	$\neg p \vee (p \vee r)$	Intro $\vee$: 4
6.	$(\neg p \vee p) \vee r$	Associativity: 5
7.	$(\neg p \vee \neg\neg p) \vee r$	Double Negation: 6
8.	$\neg(p \wedge \neg p) \vee r$	DeMorgans: 7
9.	$\underline{(p \wedge \neg p)} \rightarrow r$	Law of Implication: 8
10.	$\neg p \rightarrow r$	$\boxed{\text{Elim } \wedge\text{: 9}}$
11.	r	Modus Ponens: 4, 10

Additional Time: Predicate Logic Proofs

Problem 5b

Given $\forall x\, \exists y\, (T(x) \to S(y,x))$, we wish to prove $\exists x\, (T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\, \exists y\, (T(x) \to S(y,x))$ ________________

    2.1. $T(a)$ ________________

    Let b be arbitrary

    2.2.1. $\exists x\, (T(x) \to S(b,x))$ ________________

    2.2.2. $T(a) \to S(b,a)$ ________________

    2.2.3. $S(b,a)$ ________________

    2.2. $\forall y\, S(y,a)$ ________________

2. $T(a) \to \forall y\, S(y,a)$ ________________
3. $\exists x\, (T(x) \to \forall y\, S(y,x))$ ________________

Problem 5

Given $\forall x \, \exists y \, (T(x) \to S(y, x))$, we wish to prove $\exists x \, (T(x) \to \forall y \, S(y, x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x \, \exists y \, (T(x) \to S(y, x))$ ______ Given ______

    2.1. $T(a)$ ______

        Let b be arbitrary

        2.2.1. $\exists x \, (T(x) \to S(b, x))$ ______

        2.2.2. $T(a) \to S(b, a)$ ______

        2.2.3. $S(b, a)$ ______

    2.2. $\forall y \, S(y, a)$ ______

2. $T(a) \to \forall y \, S(y, a)$ ______

3. $\exists x \, (T(x) \to \forall y \, S(y, x))$ ______ Intro $\exists$: 2 ______

Problem 5

Given $\forall x\, \exists y\, (T(x) \to S(y,x))$, we wish to prove $\exists x\, (T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\, \exists y\, (T(x) \to S(y,x))$ ______ Given ______

    2.1. $T(a)$ ______ Assumption ______

        Let b be arbitrary

        2.2.1. $\exists x\, (T(x) \to S(b,x))$ ______

        2.2.2. $T(a) \to S(b,a)$ ______

        2.2.3. $S(b,a)$ ______

    2.2. $\forall y\, S(y,a)$ ______

2. $T(a) \to \forall y\, S(y,a)$ ______ Direct Proof: 2.2.1-2.2.3 ______
3. $\exists x\, (T(x) \to \forall y\, S(y,x))$ ______ Intro $\exists$: 2 ______

Problem 5

Given $\forall x\, \exists y\, (T(x) \rightarrow S(y, x))$, we wish to prove $\exists x\, (T(x) \rightarrow \forall y\, S(y, x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\, \exists y\, (T(x) \rightarrow S(y, x))$ ______ Given ______

    2.1. $T(a)$ ______ Assumption ______

        Let b be arbitrary

        2.2.1. $\exists x\, (T(x) \rightarrow S(b, x))$ ______ Elim $\forall$: 1 ______

        2.2.2. $T(a) \rightarrow S(b, a)$ ______

        2.2.3. $S(b, a)$ ______

    2.2. $\forall y\, S(y, a)$ ______

2. $T(a) \rightarrow \forall y\, S(y, a)$ ______ Direct Proof: 2.2.1-2.2.3 ______

3. $\exists x\, (T(x) \rightarrow \forall y\, S(y, x))$ ______ Intro $\exists$: 2 ______

Problem 5

Given $\forall x \, \exists y \, (T(x) \rightarrow S(y, x))$, we wish to prove $\exists x \, (T(x) \rightarrow \forall y \, S(y, x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x \, \exists y \, (T(x) \rightarrow S(y, x))$ ______ Given ______

    2.1. $T(a)$ ______ Assumption ______

        Let b be arbitrary

        2.2.1. $\exists x \, (T(x) \rightarrow S(b, x))$ ______ Elim $\forall$: 1 ______

        2.2.2. $T(a) \rightarrow S(b, a)$ ______ Elim $\exists$: 2.2.1 ______

        2.2.3. $S(b, a)$ ______

    2.2. $\forall y \, S(y, a)$ ______

2. $T(a) \rightarrow \forall y \, S(y, a)$ ______ Direct Proof: 2.2.1-2.2.3 ______
3. $\exists x \, (T(x) \rightarrow \forall y \, S(y, x))$ ______ Intro $\exists$: 2 ______

Problem 5

Given $\forall x\, \exists y\, (T(x) \to S(y,x))$, we wish to prove $\exists x\, (T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\, \exists y\, (T(x) \to S(y,x))$ Given

    2.1. $T(a)$ Assumption

    Let b be arbitrary

    2.2.1. $\exists x\, (T(x) \to S(b,x))$ Elim $\forall$: 1

    2.2.2. $T(a) \to S(b,a)$ Elim $\exists$: 2.2.1

    2.2.3. $S(b,a)$ Modus Ponens: 2.1, 2.2.2

    2.2. $\forall y\, S(y,a)$

2. $T(a) \to \forall y\, S(y,a)$ Direct Proof: 2.2.1-2.2.3
3. $\exists x\, (T(x) \to \forall y\, S(y,x))$ Intro $\exists$: 2

Problem 5

Given $\forall x\, \exists y\, (T(x) \to S(y,x))$, we wish to prove $\exists x\, (T(x) \to \forall y\, S(y,x))$.

The following formal proof does this, but it is missing explanations for each line. Fill in the blanks with inference rules or equivalences to apply (as well as the line numbers) to complete the proof.

1. $\forall x\, \exists y\, (T(x) \to S(y,x))$ — Given

    2.1. $T(a)$ — Assumption

        Let b be arbitrary

        2.2.1. $\exists x\, (T(x) \to S(b,x))$ — Elim $\forall$: 1

        2.2.2. $T(a) \to S(b,a)$ — Elim $\exists$: 2.2.1

        2.2.3. $S(b,a)$ — Modus Ponens: 2.1, 2.2.2

    2.2. $\forall y\, S(y,a)$ — Intro $\forall$

2. $T(a) \to \forall y\, S(y,a)$ — Direct Proof: 2.2.1-2.2.3

3. $\exists x\, (T(x) \to \forall y\, S(y,x))$ — Intro $\exists$: 2

That’s All, Folks!

Thanks for coming to section this week!
Any questions?

slides by: aruna