Quiz Section 2: Circuits, Equivalence, and Predicate Logic

Task 1 – Gates

In this problem, we will represent NAND using function syntax, meaning NAND$(p, q) \equiv \neg(p \wedge q)$.

p	q	NAND(p, q)
T	T	F
T	F	T
F	T	T
F	F	T

We can construct any gate using only NAND gates [1]. For example, we can construct NOT by using the same input for both sides of a NAND.

a) Show $\neg p \equiv$NAND(p, p) holds using a truth table.

b) Show we can express $p \wedge q$ with only NAND gates by a chain of equivalences.

c) Use what we learned in (a) plus Double Negation to write an expression for $p \vee q$. Explain in English why your expression works.

[1] this can also be done using NOR - these are "universal" gates

Task 2 – Non-equivalence

For each of the following pairs of propositions, use truth tables to determine whether they are equivalent.

Include the full truth table and state whether they are equivalent. (In principle, only one row is needed to show non-equivalence, but please turn in the entire table so that we can give partial credit in case of errors.) Your truth table must include columns for all subformulas.

a) $p \to r$ vs. $r \to p$

b) $\neg p \to (q \to r)$ vs. $q \to (p \vee r)$

c) $a \to (b \wedge c)$ vs. $(a \to b) \wedge c$

Task 3 – Equivalences

Prove that each of the following pairs of propositional formulas are equivalent using an equivalence chain.

a) $(p \to \neg p) \wedge (\neg p \to p)$ vs. F

You can also try doing this in Cozy at the following URL: https://tinyurl.com/CSE311S21a

b) $\neg p \to (q \to r)$ vs. $q \to (p \vee r)$

You can also try doing this in Cozy at the following URL: https://tinyurl.com/CSE311S21b

Task 4 – Boolean Algebra

Consider the boolean functions $F(A, B, C)$ and $G(A, B, C)$ specified by the following truth table:

A	B	C	$F(A, B, C)$	$G(A, B, C)$
1	1	1	1	1
1	1	0	1	1
1	0	1	0	1
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

a) Write the DNF and CNF expressions for $F(A, B, C)$.

b) Write the DNF and CNF expressions in boolean algebra for $G(A, B, C)$.

c) Simplify your CNF form for $G(A, B, C)$.

Task 5 – Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers and all print jobs.

a) $\exists p\,(F(p) \wedge B(p)) \rightarrow \exists j\; L(j)$

b) $(\forall j\; B(j)) \rightarrow (\exists p\, Q(p))$

c) $\exists j\;(Q(j) \wedge L(j)) \rightarrow \exists p\; F(p)$

d) $(\forall p\; B(p) \wedge \forall j\; Q(j)) \rightarrow \exists j\; L(j)$

Task 6 – Translate to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

a) Every user has access to an electronic mailbox.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.