CSE 311 Section 1

Propositional Logic

Announcements & Reminders

- Sections are Graded
  - You will be graded on section participation, so please try to come ☺
  - If you are unable to attend, please submit the completed handout to Gradescope by 5pm on Thursday.

- Section Materials
  - Handouts will be provided in at each section
  - Worksheets and sample solutions will be available on the course calendar later this evening

Tips for 311!

- Tackling challenging homework problems may feel intimidating at first but **don't go at it alone**! Find study groups, join us in office hours, book one-on-ones, and ask questions on Ed.

- Section will often be challenging and fast but valuable for your learning. **This is your time to ask lots of questions and clarify your learning**!
- Sometimes homework problems will mirror section problems, use that to your advantage!

- This class is the best time to learn how to Latex, please consider learning now as it will save you time for future courses! Feel free to come to office hours to get help with Latex!

- We have created an example latex template that you can find here: https://www.overleaf.com/read/rnxmrjvgbtrx#e59d4e

Propositions & Implications

Quick Concept Review

- **Propositions** are statements with a boolean truth value!
  - "**The AQI of Seattle is 50**" is a proposition. We know it's either true or false.
  - "**The AQI of Seattle?**" is not. Suddenly it could be hundreds of values.
  - In formal logic, we like to assign a proposition into a variable for later use.
- **Logical connectives** connect propositions to form new propositions!

$$\neg p$$

$$p \wedge q$$

$$p \vee q$$

$$p \rightarrow q$$

$$p \leftrightarrow q$$

Truth Tables

Gives us a simple way to describe how logical connectives operate

p	¬p
T	F
F	T

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Implications

Some common formulations:

- p implies q
- whenever p is true q must be true
- If p then q
- q if p
- p is sufficient for q
- p only if q
- q is necessary for p

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Vacuous truths: a false hypothesis, but true truth value

“Only if”

I attended my 8:30am class only if I woke up early

Which is equivalent?

If I woke up early then I attended my 8:30 am class

or

If I attended my 8:30 am class then I woke up early

“Only if”

I attended my 8:30am class only if I woke up early

If I woke up early then I attended my 8:30 am class

NOT Equivalent: The original statement **does not specify** what happens **when you wake up early**, you can wake up early to go play tennis in the morning!

If I attended my 8:30 am class then I woke up early

Equivalent: The original statement only **specifies exactly** what happened **when you went to your 8:30 class**, you **must** have woken up early. Nothing else could have happened for you to be attending the 8:30 class.

Problem 1

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Step 2

If p **then** q

Problem 1a – Warm Up

Steps:

1. Create propositional variables
2. Replace all propositions with created variables
3. Replace the operators

a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Step 1

p: I am lifting weights this afternoon

q: I do a warm-up exercise

Step 2

If p **then** q

Step 3

$p \rightarrow q$

Problem 1 – Translations

b) If I am cold and going to bed or I am two-years old, then I carry a blanket

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an “if then” statement

3. Replace the operators

Work on part (b) with the people around you, and then we’ll go over it together!

Problem 1 – Translations

b) If I am cold and going to bed or I am two-years old, then I carry a blanket

Step 1:
a: I am cold
b: I am going to bed
c: I am two-years old
d: I carry a blanket

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an “if then” statement

3. Replace the operators

Problem 1 – Translations

b) If I am cold and going to bed or I am two-years old, then I carry a blanket

Step 1:
a: I am cold
b: I am going to bed
c: I am two-years old
d: I carry a blanket

Step 2:
If a and b or c, then d

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an “if then” statement

3. Replace the operators

Problem 1 – Translations

b) If I am cold and going to bed or I am two-years old, then I carry a blanket

Step 1:
a: I am cold
b: I am going to bed
c: I am two-years old
d: I carry a blanket

Step 2:
If a and b or c, then d

Step 3: $[(a \wedge b) \vee c] \rightarrow d$

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an “if then” statement

3. Replace the operators

Problem 2

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don’t have homework

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don't have homework

Step 1:
a: The book has been out for a week
b: I have homework
c: I have finished reading the book

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an "if then" statement

3. Replace the operators

Problem 2 – Contrapositives

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an “if then” statement

3. Replace the operators

b) I have finished reading the book if it has been out for a week and I don’t have homework

Step 1:
a: The book has been out for a week
b: I have homework
c: I have finished reading the book

Step 2:
c if a and not b

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don't have homework

Step 1:
a: The book has been out for a week
b: I have homework
c: I have finished reading the book

Step 2:
c if a and not b

Step 2b:
if a and not b then c

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an "if then" statement

3. Replace the operators

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don't have homework

Step 1:
a: The book has been out for a week
b: I have homework
c: I have finished reading the book

Step 2:
c if a and not b

Step 2b:
if a and not b then c

Step 3: $(a \wedge \neg b) \rightarrow c.$

1. Create propositional variables
2. Replace all propositions with created variables

2b. Convert the sentence to an "if then" statement

3. Replace the operators

Problem 2 – Contrapositives

$(a \wedge \neg b) \rightarrow c.$

b) I have finished reading the book if it has been out for a week and I don't have homework

$$\neg c \rightarrow \neg(a \wedge \neg b)$$

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don't have homework

$$\neg c \rightarrow \neg(a \wedge \neg b) \equiv \neg c \rightarrow (\neg a \vee \neg\neg b)$$

Problem 2 – Contrapositives

b) I have finished reading the book if it has been out for a week and I don't have homework

$$\neg c \rightarrow \neg(a \wedge \neg b) \equiv \neg c \rightarrow (\neg a \vee \neg\neg b)$$
$$\equiv \neg c \rightarrow (\neg a \vee b)$$

Problem 2 – Contrapositives

a: The book has been out for a week
b: I have homework
c: I have finished reading the book

b) I have finished reading the book if it has been out for a week and I don't have homework

$$\neg c \rightarrow (\neg a \vee b)$$

Translation: If I have not finished reading the book then the book hasn't been out for a week or I have homework

Problem 5

(Canonical) Normal Forms

- Standard ways of translating a truth table into a proposition.
- We already did these in lecture when we translated implications into an expression only using ands, ors, and nots!
- Once you translate into one of these forms, **don't simplify your expression any further!** It often looks like you can factor variables out to make it prettier, but the whole point is to write the expression into this standardized way, so just leave it as-is ☺

DNF (OR of ANDs)

- Disjunctive Normal Form
  - OR of ANDs
  - Method:
    1. Read the TRUE rows of the truth table
    2. AND together all the variable settings in a given (true) row
    3. OR together the true rows

DNF (OR of ANDs)

1. Read the TRUE rows of the truth table
2. AND together all the variable settings in a given (true) row
3. OR together the true rows

p	*q*	*G(p,q)*
T	T	T
T	F	F
F	T	T
F	F	F

$p \wedge q$

$\neg p \wedge q$

$$G(p,q) \equiv (p \wedge q) \vee (\neg p \wedge q)$$

CNF (AND of ORs)

- Conjunctive Normal Form
  - AND of ORs
  - Method:
    1. Read the FALSE rows of the truth table
    2. OR together the negations of all the variable settings in the false row
    3. AND together the false rows

CNF (AND of ORs)

1.Read the FALSE rows of the truth table
2.OR together the negations of all the variable settings in the false row
3.AND together the false rows

p	*q*	*G(p,q)*
T	T	T
T	F	F
F	T	T
F	F	F

$\neg p \vee q$

$p \vee q$

$$G(p,q) \equiv (\neg p \vee q) \wedge (p \vee q)$$

Problem 5c

Write the CNF and DNF expression for (b)

r	q	$(r \vee q) \rightarrow (r \oplus q)$
T	T	F
T	F	T
F	T	T
F	F	T

Problem 5c

Write the CNF and DNF expression for (b)

r	q	$(r \vee q) \rightarrow (r \oplus q)$
T	T	F
T	F	T
F	T	T
F	F	T

Problem 5c

Write the CNF and DNF expression for (b)

r	q	$(r \vee q) \rightarrow (r \oplus q)$
T	T	F
T	F	T
F	T	T
F	F	T

DNFs: $(r \wedge \neg q) \vee (\neg r \wedge q) \vee (\neg r \wedge \neg q)$

Problem 5c

Write the CNF and DNF expression for (b)

r	q	(r v q) →(r ⊕ q)
T	T	F
T	F	T
F	T	T
F	F	T

CNFs: (¬r v ¬q)

That's All, Folks!

Thanks for coming to section this week!
Any questions?