CSE 311: Foundations of Computing I

Set Proof Templates

Subset Template

To prove that $A \subseteq B$ holds, fill in the $\langle\langle \ldots \rangle\rangle$ parts below:

> Let x be an arbitrary $\langle\langle$object in the domain (e.g. "integer")$\rangle\rangle$.
>
> Suppose that $x \in A$. $\langle\langle$prove $x \in B$ here$\rangle\rangle$
>
> Since x was arbitrary, we have proven, by the definition of subset, that $A \subseteq B$.

Set Equality Template

If A and B are each defined in terms of set operations (e.g., $\cup$, $\cap$, $\bar{\cdot}$, and $\setminus$), fill in the $\langle\langle \ldots \rangle\rangle$ parts below to prove that $A = B$:

> Let x be an arbitrary $\langle\langle$object in the domain (e.g. "integer")$\rangle\rangle$.
>
> The stated biconditional holds since

$$\begin{aligned} x \in A &\equiv \langle\langle \text{replace } \cup,\ \cap,\ \ldots \text{with } \vee,\ \wedge,\ \ldots \rangle\rangle \\ &\equiv \langle\langle \text{apply equivalences from Propositional Logic} \rangle\rangle \\ &\equiv \langle\langle \text{replace } \vee,\ \wedge,\ \ldots \text{with } \cup,\ \cap,\ \ldots \rangle\rangle \\ &\equiv x \in B \end{aligned}$$

> Since x was arbitrary, we have proven, by the definition of set equality, that $A = B$.

The explanations for replacing set operators with logical operators and vice versa are "def of $\langle\langle$logical operator$\rangle\rangle$" (e.g., "def of $\cup$"). The explanations for Propositional Logic equivalences are the names of those equivalences on the Equivalences reference sheet (e.g., "Double Negation")