CSE 311: Foundations of Computing I

English Proof Cheatsheet

Propositional Logic: Workhorse Rules

Rule	English
Given $\dfrac{A \text{ is given}}{\therefore\ A}$	We are given that A holds. (*Write this where A is <u>first used</u>. Not at the start of the proof.*)
Modus Ponens $\dfrac{A \quad A \to B}{\therefore\ B}$	Since A holds, we can see that B must hold.
Direct Proof $\dfrac{A \Rightarrow B}{\therefore\ A \to B}$	Suppose A holds. …Thus, B holds. (end of paragraph) (*That is the English for the subproof. Direct Proof itself is skipped.*)
Elim $\wedge$ $\dfrac{A \wedge B}{\therefore\ A \qquad B}$	*Skip* (*In English, knowing A and B is the same as knowing $A \wedge B$.*)
Intro $\wedge$ $\dfrac{A \quad B}{\therefore\ A \wedge B}$	*Skip*
Proof By Cases $\dfrac{A \vee B \quad A \to C \quad B \to C}{\therefore\ C}$	Since we know that either A or B is true, it follows that C holds.
Intro $\vee$ $\dfrac{A}{\therefore\ A \vee B \qquad B \vee A}$	Since A holds, we know that $A \vee B$ holds. *(Avoid writing "$A \vee B$" if at all possible, e.g., if we can instead say "Since A holds, we can see that the claim holds.")*

Propositional Logic: Alternative Rules

Tautology

$$\frac{A \equiv \top}{\therefore \quad A}$$

We know that A holds since it is a tautology

(*Skip this altogether if it is $A \lor \neg A$, i.e., Excluded Middle.*)

Equivalent

$$\frac{A \equiv B \qquad B}{\therefore \qquad A}$$

Equivalently, we have shown that A holds.

Principium Contradictionis

$$\frac{A \quad \neg A}{\therefore \quad \bot}$$

[[Proof of A.]]

[[Proof of $\neg A$.]] This contracts the fact that A holds.

Reductio Ad Absurdum

$$\frac{A \Rightarrow \bot}{\therefore \quad \neg A}$$

Suppose A holds. ... This is a contradiction. Thus, our assumption that A holds must have been false.

Ex Falso Quodlibet

$$\frac{\bot}{\therefore \; A}$$

Since false is true, anything is true. In particular, A is true.

Ad Litteram Verum

$$\frac{}{\therefore \; \top}$$

Skip

Predicate Logic

Elim $\forall$

$$\frac{\forall x, P(x)}{\therefore\ P(a) \text{ for } \textit{any} \text{ object } a}$$

Since $P(x)$ holds for any x, we know that $P(a)$ holds

Intro $\forall$

$$\frac{\text{Let } a \text{ be arbitrary} \Rightarrow P(a)}{\therefore\quad \forall x, P(x)}$$

Let a be an arbitrary integer. ... Since a was arbitrary, we have shown that $P(a)$ holds for every integer a.

(*That is the English for the subproof. Intro $\forall$ itself is skipped.*)

Elim $\exists$

$$\frac{\exists x, P(x)}{\therefore\ P(c) \text{ for a } \textit{new} \text{ name } c}$$

$P(c)$ holds for some integer c.

(Avoid writing "$\exists x, P(x)$". Instead, immediately eliminate the $\exists$ and write down $P(c)$ for some new variable c.)

Intro $\exists$

$$\frac{P(c) \text{ for some } c}{\therefore\quad \exists x, P(x)}$$

We have shown that there exists an x such that $P(x)$ holds.

(Avoid writing "$\exists x, P(x)$" if at all possible, e.g., if we can instead say "Thus, we can see that the claim holds.")

Theorems and Definitions

Cite T

$$\frac{}{\therefore\ \forall x, P(x) \to Q(x)}$$

Theorem T tells us that $P(x)$ implies $Q(x)$, for all x.

(Use Apply instead whenever possible. It sounds more natural.)

Apply T

$$\frac{P(c) \text{ for some } c}{\therefore\quad Q(c)}$$

Since $P(c)$ holds, it follows from Theorem T that $Q(c)$ holds.

Def of f

$$\frac{P}{\therefore\ P[f(c) \,/\, (\exists y, c = g(y))]}$$

This can be written equivalently as $P[f(c) \,/\, (\exists y, c = g(y))$, by the definition of f.

Undef f

$$\frac{P[f(c) \,/\, (\exists y, c = g(y))]}{\therefore\quad P}$$

This can be written equivalently as P, by the definition of f.