Section 09: Models of Computation

1. Regular Expressions

Note: this question was also on the Section 8 supplemental handout.

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3 without leading zeroes.

(c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".

(d) Write a regular expression that matches all binary strings that do not have any consecutive 0's or 1's.

(e) Write a regular expression that matches all binary strings of the form $1^k y$, where $k \geq 1$ and $y \in \{0,1\}^*$ has at least k 1's.

2. CFGs

Write a context-free grammar to match each of these languages.

(a) All binary strings that start with 11.

(b) All binary strings that contain at most one 1.

(c) All strings over 0, 1, 2 with the same number of 1s and 0s and exactly one 2.

3. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1,2,3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

4. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

5. All The Models!

Construct a valid regular expression, CFG, and DFA for the following languages.

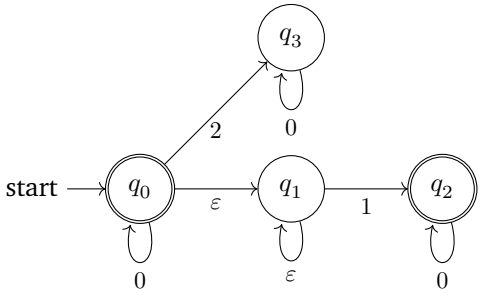
(a) All strings whose base-6 representation is divisible by 3 (leading zeros are ok). Let $\Sigma = \{0, 1, 2, 3, 4, 5\}$.

(b) All binary strings of 0s capped by a 1 on either side.

6. NFAs, Stage 1

Note: This content will be covered in lecture on Nov 21st and 24th. Here is a video walkthrough posted in place of the section that would be on Thanksgiving week! Check the course calendar for slides.

(a) What language does the following NFA accept?



(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

(c) Create an NFA for the language "all binary strings that start with 101 or 011".

(d) Create an NFA for the language "all strings composed of 0 or more occurrences of 101 only".

7. NFAs, Stage 2

Note: This content will be covered in lecture on Nov 21st and 24th. Here is a video walkthrough posted in place of the section that would be on Thanksgiving week! Check the course calendar for slides.

Hint: Note that we already found the regex for these in Question 1. Referencing your answers for that can be helpful!

(a) Create an NFA for the language "all base 3 numbers without leading zeros that are divisible by 3".

(b) Create an NFA for the language "all binary strings that contain the substring '111', but not the substring '000'".

(c) Create an NFA for the language "all binary strings that do not have any consecutive 0's or 1's".

8. DFAs & Minimization

Note: We will not test you on minimization, although you may optionally read the extra slides and do this problem for fun

(a) Convert the NFA from 6a to a DFA, then minimize it.

(b) Minimize the following DFA: