Section 08: Regular Expressions, Graphs, Functions

1. Regular Expressions

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

(c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".

(d) Write a regular expression that matches all binary strings that do not have any consecutive 0's or 1's.

(e) Write a regular expression that matches all binary strings of the form $1^k y$, where $k \geq 1$ and $y \in \{0,1\}^*$ has at least k 1's.

2. Graphs

For each graph $G = (V, E)$, draw the graph. Additionally, determine if the graph has a cycle or simple cycle.

(a) $G = (\{1,2,3\}, \{(1,2),(2,3),(3,2),(1,1)\})$

(b) $G = (\{1,2,3,4\}, \{(1,2),(1,3),(2,4),(3,4)\})$

3. Classifying Functions

For each of these functions, state whether it is one-to-one, onto, both, or neither.

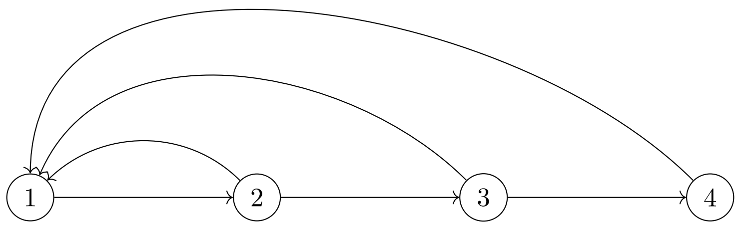
(a) $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

(c) $f : \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = x^2$, where $\mathbb{R}^+ = \{x : x \in \mathbb{R} \wedge x \geq 0\}$, i.e., the set of non-negative real numbers.

4. More Graphs

The following questions in this section are about the following graph:



(a) What are V and E in the graph above?

(b) Find a path in the graph that is not simple.

(c) Find a simple path in the graph.

(d) Find a cycle in the graph that is not simple.

(e) Find a simple cycle in the graph.

5. Onto and One-to-One

(a) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is onto but not one-to-one. Be specific.

(b) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is one-to-one but not onto. Be specific.

6. A Bijection Proof

Let A be the set of negative integers, i.e., $A = \{-1, -2, -3, ...\}$; let B be the set of integers at least 10, i.e., $B = \{10, 11, 12, 13, ...\}$. Show that $f : A \to B$ defined by $f(x) = |x| + 9$ is a bijection.

You may use these facts:

- for negative numbers x, y: $|x| = |y| \to x = y$
- for negative numbers $|x| = -x$