Bijection

One-to-one (aka injection)
A function f is one-to-one iff $\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)
A function $f: A \rightarrow B$ is onto iff $\forall b \in B \exists a \in A(b = f(a))$

Bijection
A function $f: A \rightarrow B$ is a bijection iff f is one-to-one and onto

A bijection maps every element of the domain to **exactly** one element of the co-domain, and every element of the domain to **exactly** one element of the domain.

52

Proof that [0,1) is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	…
$f(0)$	0.	3	3	3	3	3	3	3	3	…
$f(1)$	0.	2	7	2	7	2	8	5	4	…
$f(2)$	0.	1	4	1	5	9	2	6	5	…
$f(3)$	0.	2	2	2	2	2	2	2	2	…
$f(4)$	0.	1	2	3	4	5	6	7	8	…
$f(5)$	0.	9	8	7	6	5	4	3	2	…
$f(6)$	0.	8	2	7	6	4	5	7	4	…
$f(7)$	0.	5	9	4	2	7	5	1	7	…
…	…	…	…	…	…	…	…	…	…	…

24

To Show S is uncountable

Suppose, for the sake of contradiction, that S is countable. Then by definition, S can be put in bijection with the natural numbers.

Visualize that bijection as a table: the i^{th} row of the table is the i^{th} element of S in the bijection. Each column of the table contains a <>. *Drawing a doodle of the table is usually helpful here*.

Observe that since elements of S are made up of a sequence of infinitely many <>, the table is infinite in both directions.

We now construct an element e of S which is not in the table.

Describe a construction rule, probably by flipping. Notation helps here, "the i^{th} <> of e is …"

50

To Show S is uncountable

Explain why e really is in S, you'll probably want to mention going on infinitely, and something like: this is an infinitely-long sequence of integers, but that is just the decimal expansion of a real number between 0 and 1.

Note that e cannot be the element in row i of the table for any i, as … (*refer back to flipping rule to argue you aren't equal to row i*).

Thus e is an element of S not on the table. But the table represents a bijection between S and $\mathbb{N}$. That's a contradiction! So S must be uncountably infinite.

51