

## Regular Expressions

### Basis:

$\varepsilon$  is a regular expression. The empty string itself matches the pattern (and nothing else does).

$\emptyset$  is a regular expression. No strings match this pattern.

$a$  is a regular expression, for any  $a \in \Sigma$  (i.e. any character). The character itself matching this pattern.

### Recursive

If  $A, B$  are regular expressions then  $(A \cup B)$  is a regular expression matched by any string that matches  $A$  or that matches  $B$  [or both].

If  $A, B$  are regular expressions then  $AB$  is a regular expression. matched by any string  $x$  such that  $x = yz$ ,  $y$  matches  $A$  and  $z$  matches  $B$ .

If  $A$  is a regular expression, then  $A^*$  is a regular expression. matched by any string that can be divided into 0 or more strings that match  $A$ .

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## More Examples

$$(0^*1^*)^*$$

$$0^*1^*$$

$$(0 \cup 1)^*(00 \cup 11)^*(0 \cup 1)^*$$

$$(00 \cup 11)^*$$

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## More Practice

You can also go the other way

Write a regular expression for "the set of all binary strings of odd length"

Write a regular expression for "the set of all binary strings with at most two ones"

Write a regular expression for "strings that don't contain 00"

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## More Practice

All binary strings with a 1 in the third position. (index from 1)

All binary strings with a 1 in the third position from the end (with length at least three).

All binary strings with an even number of 0s or exactly one 1.

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