One-to-one proofs - structure

It's a for-all statement! We know how to prove it.

Let $f: \mathbb{Z} \to \mathbb{Z}$ be the function given by $f(x) = x + 5$.

Claim: f is one-to-one

Proof:

What's the outline? What do we introduce, what do we assume, what's our target?

One-to-one (aka injection)

A function f is one-to-one iff
$\forall a \forall b (f(a) = f(b) \to a = b)$

Onto proofs - structure

It's a for-all statement, with an exists inside! We know how to prove it.

Let $f: \mathbb{Z} \to \mathbb{Z}$ be the function given by $f(x) = x + 5$.

Claim: f is onto

Proof:

What's the outline? What do we introduce, what do we assume, what's our target?

Onto (aka surjection)

A function $f: A \to B$ is onto iff
$\forall b \in B \exists a \in A (b = f(a))$

Bijection

One-to-one (aka injection)

A function f is one-to-one iff
$\forall a \forall b (f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff
$\forall b \in B \exists a \in A (b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff
f is one-to-one and onto

A bijection maps every element of the domain to exactly one element of the co-domain, and every element of the codomain to exactly one element of the domain.

Some infinite sets - comparisons

Two sets A, B have the same size (same cardinality) if and only if there is a bijection $f: A \rightarrow B$

Let's compare the sizes of: $\mathbb{N}$, $\mathbb{Z}$, $\{x : x$ is an even integer$\}$