Recursive Definitions of Sets

Q1: What is this set?

Basis Step: $6 \in S, 15 \in S$

Recursive Step: If $x, y \in S$ then $x + y \in S$

Q2: Write a recursive definition for the set of powers of 3 $\{1,3,9,27, \ldots\}$

Basis Step:

Recursive Step:

Structural Induction Template

1. Define $P()$ State that you will show $P(x)$ holds for all $x \in S$ and that your proof is by structural induction.
2. Base Case: Show $P(b)$
   [Do that for every b in the basis step of defining S]
3. Inductive Hypothesis: Suppose $P(x)$
   [Do that for every x listed as already in S in the recursive rules].
4. Inductive Step: Show $P()$ holds for the "new elements."
   [You will need a separate step for every element created by the recursive rules].
5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

Strings

ε is "the empty string"

The string with 0 characters – `""` in Java (not `null!`)

Σ^*:

Basis: $\varepsilon \in \Sigma^*$.

Recursive: If $w \in \Sigma^*$ and $a \in \Sigma$ then $wa \in \Sigma^*$

wa means the string of w with the character a appended.

You'll also see $w \cdot a$ (a $\cdot$ to mean "concatenate" i.e. + in Java)

Length:

len(ε)=0;

len(wa)=len(w)+1 for $w \in \Sigma^*$, $a \in \Sigma$

Reversal:

$\varepsilon^R = \varepsilon$;

$(wa)^R = aw^R$ for $w \in \Sigma^*$, $a \in \Sigma$

Concatenation

$x \cdot \varepsilon = x$ for all $x \in \Sigma^*$;

$x \cdot (wa) = (x \cdot w)a$ for $w \in \Sigma^*, a \in \Sigma$

Number of c's in a string

$\#_c(\varepsilon) = 0$

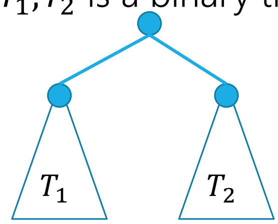
$\#_c(wc) = \#_c(w) + 1$ for $w \in \Sigma^*$;

$\#_c(wa) = \#_c(w)$ for $w \in \Sigma^*, a \in \Sigma \setminus \{c\}$.

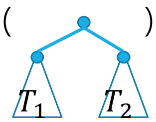
Binary Trees

Basis: A single node is a rooted binary tree.

Recursive Step: If T_1 and T_2 are rooted binary trees with roots r_1 and r_2, then a tree rooted at a new node, with children r_1, r_2 is a binary tree.

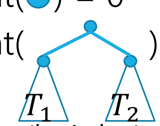


size( )=1

size() =

size(T_1) + size(T_2) + 1

height( ) = 0

height() =

1+max(height(T_1),height(T_2))