More Induction (goal)

Induction doesn't **only** work for code!

Show that $\sum_{i=0}^{n} 2^i = 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$.

Induction on Primes (revised)

Let $P(n)$ be "n can be written as a product of primes."

We show $P(n)$ for all integers $n \geq 2$ by induction on n.

Base Case ($n = 2$): 2 is a product of just itself. Since 2 is prime, it is written as a product of primes.

Inductive Hypothesis:

Inductive Step:
Case 1, $k + 1$ is prime: then $k + 1$ is automatically written as a product of primes.

Case 2, $k + 1$ is composite:

Therefore $P(k + 1)$.

$P(n)$ holds for all $n \geq 2$ by the principle of induction.

Making Strong Induction Proofs Pretty

All of our strong induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.
2. Base Case: Show $P(b)$ i.e. show the base case
3. Inductive Hypothesis: Suppose $\mathrm{P(b)} \wedge \cdots \wedge P(k)$ for an arbitrary $k \geq b$.
4. Inductive Step: Show $P(k+1)$ (i.e. get $[\mathrm{P(b)} \wedge \cdots \wedge P(k)] \rightarrow P(k+1)$)
5. Conclude by saying $P(n)$ is true for all $n \geq b$ by the principle of induction.

Stamp Collection (attempt)

Define $P(n)$ "I can make n cents of stamps with just 4 and 5 cent stamps."

We prove $P(n)$ is true for all integers $n \geq 12$ by induction on n.

Base Case:

12 cents can be made with three 4 cent stamps.

Inductive Hypothesis Suppose [maybe some other stuff and] $P(k)$, for an arbitrary $k \geq 12$.

Inductive Step:

We want to make $k+1$ cents of stamps. By IH we can make $k-3$ cents exactly with stamps. Adding another 4 cent stamp gives exactly $k+1$ cents.