

## How do we know recursion works?

```
//Assume i is a nonnegative integer
//returns 2^i.
public int CalculatesTwoToTheI(int i){
    if(i == 0)
        return 1;
    else
        return 2*CaclulatesTwoToTheI(i-1);
}
```

Why does `CalculatesTwoToTheI(4)` calculate  $2^4$ ?  
Convince the people around you!

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## Making Induction Proofs Pretty

All of our induction proofs will come in 5 easy(?) steps!

1. Define  $P(n)$ . State that your proof is by induction on  $n$ .
2. Show  $P(0)$  i.e. show the base case
3. Suppose  $P(k)$  for an arbitrary  $k$ .
4. Show  $P(k + 1)$  (i.e. get  $P(k) \rightarrow P(k + 1)$ )
5. Conclude by saying  $P(n)$  is true for all  $n$  by induction.

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## The Principle of Induction (formally)

Principle of  
Induction

If you know:  $P(0); \forall k(P(k) \rightarrow P(k + 1))$

You can conclude:  $\forall n(P(n))$

Informally: if you knock over one domino, and every domino knocks over the next one, then all your dominoes fell over.

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## Making Induction Proofs Pretty

Let  $P(n)$  be the predicate " $\dots$ "  
We prove  $P(n)$  holds for all natural numbers  $n$  by induction on  $n$ .

**Base Case** ( $n = 0$ )

**Inductive Hypothesis:**

**Inductive Step:**

Therefore

by the principle of induction.

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