

```

public int Mystery(int m, int n){
    if(m<n){
        int temp = m;
        m=n;
        n=temp;
    }
    while(n != 0) {
        int rem = m % n;
        m=n;
        n=rem;
    }
    return m;
}

```

## Finding the inverse...

$$\begin{aligned}
 \gcd(26,7) &= \gcd(7, 26\%7) = \gcd(7,5) \\
 &= \gcd(5, 7\%5) = \gcd(5,2) \\
 &= \gcd(2, 5\%2) = \gcd(2, 1) \\
 &= \gcd(1, 2\%1) = \gcd(1,0) = 1.
 \end{aligned}$$

$$26 = 3 \cdot 7 + 5; 5 = 26 - 3 \cdot 7$$

$$7 = 5 \cdot 1 + 2; 2 = 7 - 5 \cdot 1$$

$$5 = 2 \cdot 2 + 1; 1 = 5 - 2 \cdot 2$$

## An application of all of this modular arithmetic

Amazon chooses random 512-bit (or 1024-bit) prime numbers  $p, q$  and an exponent  $e$  (often about 60,000).

Amazon calculates  $n = pq$ . They tell your computer  $(n, e)$  (not  $p, q$ )

You want to send Amazon your credit card number  $a$ .

You compute  $C = a^e \% n$  and send Amazon  $C$ .

Amazon computes  $d$ , the multiplicative inverse of  $e \pmod{[p-1][q-1]}$

Amazon finds  $C^d \% n$

Fact:  $a = C^d \% n$  as long as  $0 < a < n$  and  $p \nmid a$  and  $q \nmid a$

## Fast Exponentiation Algorithm

What if  $e$  isn't exactly a power of 2?

Step 1: Write  $e$  in binary.

Step 2: Find  $a^c \% n$  for  $c$  every power of 2 up to  $e$ .

Step 3: calculate  $a^e$  by multiplying  $a^c$  for all  $c$  where binary expansion of  $e$  had a 1.