More proofs

Show that if $a \equiv b (mod\ n)$ and $c \equiv d\ (mod\ n)$ then $ac \equiv bd\ (mod\ n)$.

Step 1: What do the words mean?

Step 2: What does the statement as a whole say?

Step 3: Where do we start?

Step 4: What's our target?

Step 5: Now prove it.

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Warm up

Equivalence in modular arithmetic

Let $a \in \mathbb{Z}, b \in \mathbb{Z}, n \in \mathbb{Z}$ and $n > 0$.
We say $a \equiv b\ (mod\ n)$ if and only if $n|(b - a)$

Show that $a \equiv b\ (mod\ n)$ if and only if $b \equiv a (mod\ n)$

How do you prove an if and only if?

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```
public int Mystery(int m, int n){
      if(m<n){
            int temp = m;
            m=n;
            n=temp;
      }
      while(n != 0) {
            int rem = m % n;
            m=n;
            n=rem;
      }
      return m;
}
```

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So…what's it good for?

Suppose I want to solve $7x \equiv 3(mod\ n)$

Just multiply both sides by $\frac{1}{7}$…

Oh wait. We want a number to multiply by 7 to get 1.

If the gcd(7,n) = 1

Then $s \cdot 7 + tn = 1$, so $7s - 1 = -tn$ i.e. $n|(7s - 1)$ so $7s \equiv 1(mod\ n)$.

So the s from Bézout's Theorem is what we should multiply by!

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