Trying a direct proof (setup)

$\forall a(\texttt{Even}(a^2) \rightarrow \texttt{Even}(a))$

Proof By Contradiction (setup)

Claim: $\sqrt{2}$ is irrational (i.e. not rational).

Proof:

Rational

A real number x is rational if (and only if) there exist integers p and q, with $q \neq 0$ such that $x = p/q$.

$\texttt{Rational}(x) \coloneqq \exists p \exists q(\texttt{Integer}(p) \wedge \texttt{Integer}(q) \wedge (x = p/q) \wedge q \neq 0)$

What's the difference?

What's the difference between proof by contrapositive and proof by contradiction?

Show $p \to q$	Proof by contradiction	Proof by contrapositive
Starting Point	$\neg(p \to q) \equiv (p \land \neg q)$	$\neg q$
Target	Something false	$\neg p$

Show p	Proof by contradiction	Proof by contrapositive
Starting Point	$\neg p$	---
Target	Something false	---

Another Proof By Contradiction (setup)

Claim: There are infinitely many primes.

Proof: