Try it! (setup)

Given: $p \vee q$, $(r \wedge s) \rightarrow \neg q$, r.
Show: $s \rightarrow p$

Eliminate ∧: $\dfrac{A \wedge B}{\therefore\ A, B}$

Eliminate ∨: $\dfrac{A \vee B, \neg A}{\therefore\ B}$

Intro ∧: $\dfrac{A; B}{\therefore\ A \wedge B}$

Intro ∨: $\dfrac{A}{\therefore\ A \vee B, B \vee A}$

Direct Proof rule: $\dfrac{A \Rightarrow B}{A \rightarrow B}$

Modus Ponens: $\dfrac{P \rightarrow Q; P}{\therefore\ Q}$

You can still use all the propositional logic equivalences too!

Inference Rules for Proofs with Quantifiers

We've done symbolic proofs with propositional logic.

To include predicate logic, we'll need some rules about how to use quantifiers.

Eliminate ∀: $\dfrac{\forall x\, P(x)}{\therefore\ P(a) \text{ for any } a}$

Intro ∃: $\dfrac{P(c) \text{ for some } c}{\therefore\ \exists x\, P(x)}$

Intro ∀: $\dfrac{P(a); a \text{ is arbitrary}}{\therefore\ \forall x\, P(x)}$

Eliminate ∃: $\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for a fresh } c}$

Let's see a good example, then come back to those "arbitrary" and "fresh" conditions.

Arbitrary Practice (1)

In section, you said: $[\exists y \forall x\, P(x, y)] \rightarrow [\forall x \exists y\, P(x, y)]$. Let's prove it!!

Find The Bug

Let your domain of discourse be integers.
We claim that given $\forall x \exists y$ $\texttt{Greater}(y, x)$, we can conclude $\exists y \forall x$ $\texttt{Greater}(y, x)$
Where $\texttt{Greater}(y, x)$ means $y > x$

1. $\forall x \exists y$ $\texttt{Greater}(y, x)$ — Given
2. Let a be an arbitrary integer — --
3. $\exists y$ $\texttt{Greater}(y, a)$ — Elim ∀ (1)
4. $\texttt{Greater}(b, a)$ — Elim ∃ (2)
5. $\forall x$ $\texttt{Greater}(b, x)$ — Intro ∀ (4)
6. $\exists y \forall x$ $\texttt{Greater}(y, x)$ — Intro ∃ (5)