

1. The statement is true for every x , we just want to put a name on it.
 $\forall x (p(x) \wedge q(x))$ means "for every x in our domain, $p(x)$ and $q(x)$ both evaluate to true."

Universal Quantifier

" $\forall x$ "

"for each x ", "for every x ", "for all x " are common translations

Remember: upside-down-A for All.

2. There's some x out there that works, (but I might not know which it is, so I'm using a variable).

$\exists x (p(x) \wedge q(x))$ means "there is an x in our domain, $p(x)$ and $q(x)$ are both true."

Existential Quantifier

" $\exists x$ "

"there is an x ", "there exists an x ", "for some x " are common translations

Remember: backwards-E for Exists.

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To "limit" variables to a portion of your domain of discourse under a universal quantifier add a hypothesis to an implication.

To "limit" variables to a portion of your domain of discourse under an existential quantifier AND the limitation together with the rest of the statement.

To negate an expression with a quantifier
 1. Switch the quantifier (\forall becomes \exists , \exists becomes \forall)
 2. Negate the expression inside

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Negation

Translate these sentences to predicate logic, then negate them.

All cats have nine lives.

All dogs love every person.

There is a cat that loves someone.

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Nested Quantifiers

Let our domain of discourse be $\{A, B, C, D, E\}$

And our proposition $P(x, y)$ be given by the table.

What should we look for in the table?

$$\exists x \forall y P(x, y)$$

$$\forall x \exists y P(x, y)$$

	y				
$P(x, y)$	A	B	C	D	E
A	T	T	T	T	T
B	T	F	F	T	F
C	F	T	F	F	F
D	F	F	F	F	T
E	F	F	F	T	F

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