Our First Proof

$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b)$	$\equiv (a \wedge b) \vee [(\neg a \wedge b) \vee (\neg a \wedge \neg b)]$	Associative
	$\equiv (a \wedge b) \vee [\neg a \wedge (b \vee \neg b)]$	Distributive
	$\equiv (a \wedge b) \vee [\neg a \wedge \mathrm{T}]$	Negation
	$\equiv (a \wedge b) \vee [\neg a]$	Identity
	$\equiv [\neg a] \vee (a \wedge b)$	Commutative
	$\equiv (\neg a \vee a) \wedge (\neg a \vee b)$	Distributive
	$\equiv (a \vee \neg a) \wedge (\neg a \vee b)$	Commutative
	$\equiv \mathrm{T} \wedge (\neg a \vee b)$	Negation
	$\equiv (\neg a \vee b) \wedge \mathrm{T}$	Commutative
	$\equiv (\neg a \vee b)$	Identity

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Properties of Logical Connectives

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \to q \equiv \neg p \vee q$
- Contrapositive
  - $p \to q \equiv \neg q \to \neg p$

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Converse, Contrapositive

Implication:

$p \rightarrow q$

Converse:

$q \rightarrow p$

Contrapositive:

$\neg q \rightarrow \neg p$

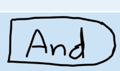
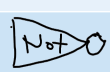
Inverse:

$\neg p \rightarrow \neg q$

An implication and its contrapositive have the same truth value!

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Meet Boolean Algebra

Name	Variables	"True/False"	"And"	"Or"	"Not"	Implication
Java Code	`boolean b`	`true,false`	`&&`	`||`	`!`	No special symbol
Propositional Logic	$"p,q,r"$	`T, F`	$\wedge$	$\vee$	$\neg$	$\rightarrow$
Circuits	Wires	1, 0				No special symbol
Boolean Algebra	a, b, c	1,0	$\cdot$ ("multiplication")	$+$ ("addition")	$'$ (apostrophe after variable)	No special symbol

Propositional logic

$(p \wedge q \wedge r) \vee s \vee \neg t$

Boolean Algebra

$pqr + s + t'$